From Integers to Factoring – Using Algebra Tiles in the GED Classroom

2019 GED Annual Conference

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Evidence-Based Practices for the Math Classroom

Instructional Element	Recommended Practices
Curriculum Design	• Ensure mathematics curriculum is based on challenging content • Ensure curriculum is standards based • Clearly identify skills, concepts and knowledge to be mastered • Ensure that the mathematics curriculum is vertically and horizontally articulated

Professional Development for Teachers	• Provide professional development which focuses on:
	  ○ Knowing/understanding standards
	  ○ Using standards as a basis for instructional planning
	  ○ Teaching using best practices
	  ○ Multiple approaches to assessment
	• Develop/provide instructional support materials such as curriculum maps and pacing guides and provide math coaches

Technology	• Provide professional development on the use of instructional technology tools • Provide student access to a variety of technology tools • Integrate the use of technology across all mathematics curricula and courses
Manipulatives	• Use manipulatives to develop understanding of mathematical concepts • Use manipulatives to demonstrate word problems • Ensure use of manipulatives is aligned with underlying math concepts
Instructional Strategies	• Focus lessons on specific concept/skills that are standards based • Differentiate instruction through flexible grouping, individualizing lessons, compacting, using tiered assignments, and varying question levels • Ensure that instructional activities are learner-centered and emphasize inquiry/problem-solving • Use experience and prior knowledge as a basis for building new knowledge • Use cooperative learning strategies and make real life connections • Use scaffolding to make connections to concepts, procedures and understanding • Ask probing questions which require students to justify their responses • Emphasize the development of basic computational skills

Algebraic Thinking

What is algebraic thinking? When do you think students first begin to algebraically think? Algebraic thinking is very simply the ideas of algebra and the skill of being able to logically think. Algebraic ideas include patterns, variables, expressions, equations, and functions. These are the building blocks of algebraic thinking. Translating words into symbols is similar to modeling a situation using an equation and variables. Students need to know that it is through algebraic equations and inequalities that they can represent a quantitative relationship between two or more objects.

Teaching algebra in today's classroom is not as much about manipulating letters and numbers in equations that don't make sense, but rather understanding operations and processes. Before beginning the process of teaching algebra, be sure that students understand the basics. The key prerequisites for students to be successful in the study of algebra are to first understand the:

- concept of variables; and
- concept of relations and functions.

When teaching algebra, it's important to use practical experiences that go beyond the mere computation required by equations. When developing practice activities in the algebra classroom, be sure that you:

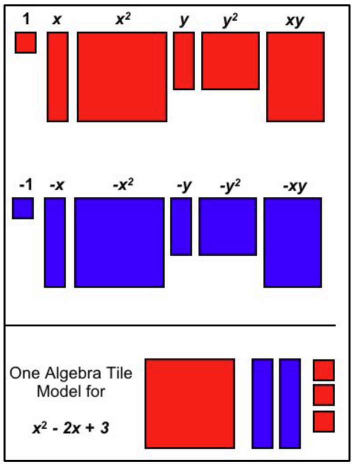
- Develop processes/procedures for students to use when approaching algebraic tasks
- Create exercises that highlight the critical attributes related to the skill or concept being taught
- Provide opportunities for students to verbalize about the task and predict what type of answer is expected
- Offer opportunities for students to discuss and write responses to questions dealing with key concepts being learned
- Select exercises that anticipate future skills to be learned
- Design exercises that integrate a number of ideas to reinforce prior learning as well as current, and future concepts

As students learn algebra, they need to develop different procedures to use. Being able to recognize a pattern is an important critical thinking skill in solving certain algebraic problems.

- **Finding** patterns involves looking for regular features of a situation that repeats.
- **Describing** patterns involves communicating the regularity in words or in a mathematically concise way that other people can understand.
- **Explaining** patterns involves thinking about why the pattern continues forever, even if one has not exhaustively looked at each one.
- **Predicting** with patterns involves using your description to predict pieces of the situation that are not given.

Algebra Tiles – Algebra for All

Algebra for all is possible. The problem is that often algebra is initially taught by using rote memorization of rules or short cuts resulting in students not understanding the “why” of these abstract concepts.



What Are Algebra Tiles?
Algebra tiles are rectangular shapes that provide area models of variables and integers. They can consist of “*x*” sets and “*y*” sets or just “x” sets. Sets consist of two different colors to represent both positive and negative terms.

Why Use Algebra Tiles?
Manipulating algebra tiles combines an algebraic and a geometric approach to algebraic concepts. Algebra tiles are used to build concrete area representations of abstract algebraic concepts. The concrete representations help students become comfortable with using symbols to represent algebraic concepts. Sequencing instruction from the concrete level to the representational level, and finally to the abstract or symbolic level helps a broader group of students better understand and solve algebraic problems. Algebra tiles provide a frame of reference to students who are not abstract thinkers.

How to Use Algebra Tiles
There are many different pre-algebra and algebra concepts that can be introduced and developed using algebra tiles:

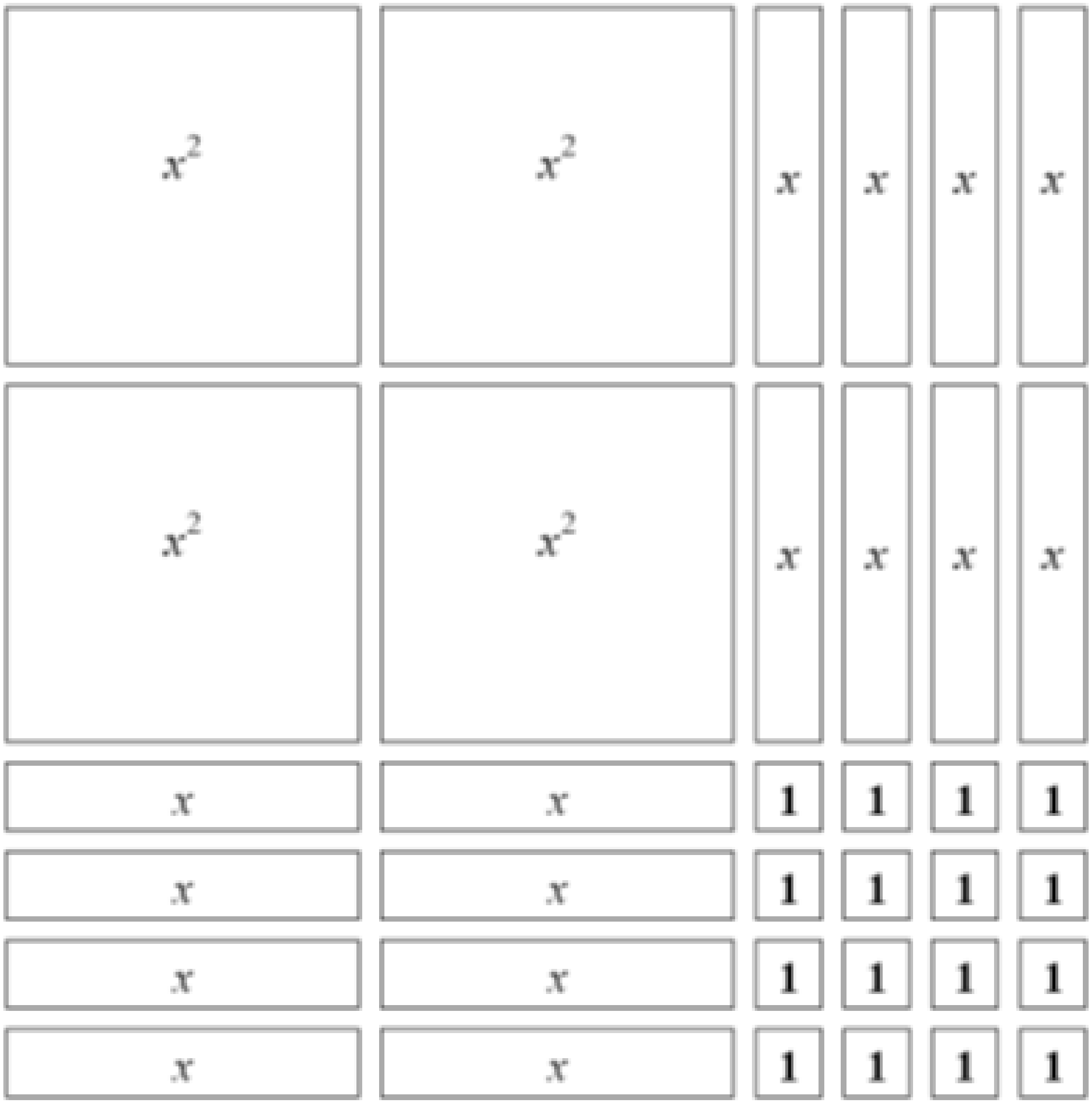
- Adding, subtracting, multiplying, and dividing integers
- Modeling algebraic expressions and combing like terms
- Using the distributive property
- Solving linear equations using addition, subtraction, multiplication, or division
- Solving general linear equations involving two or more steps
- Multiplying a monomial by a monomial, a binomial by a monomial, or a binomial by a binomial
- Factoring quadratic trinomials or the difference of two square
- Completing the square

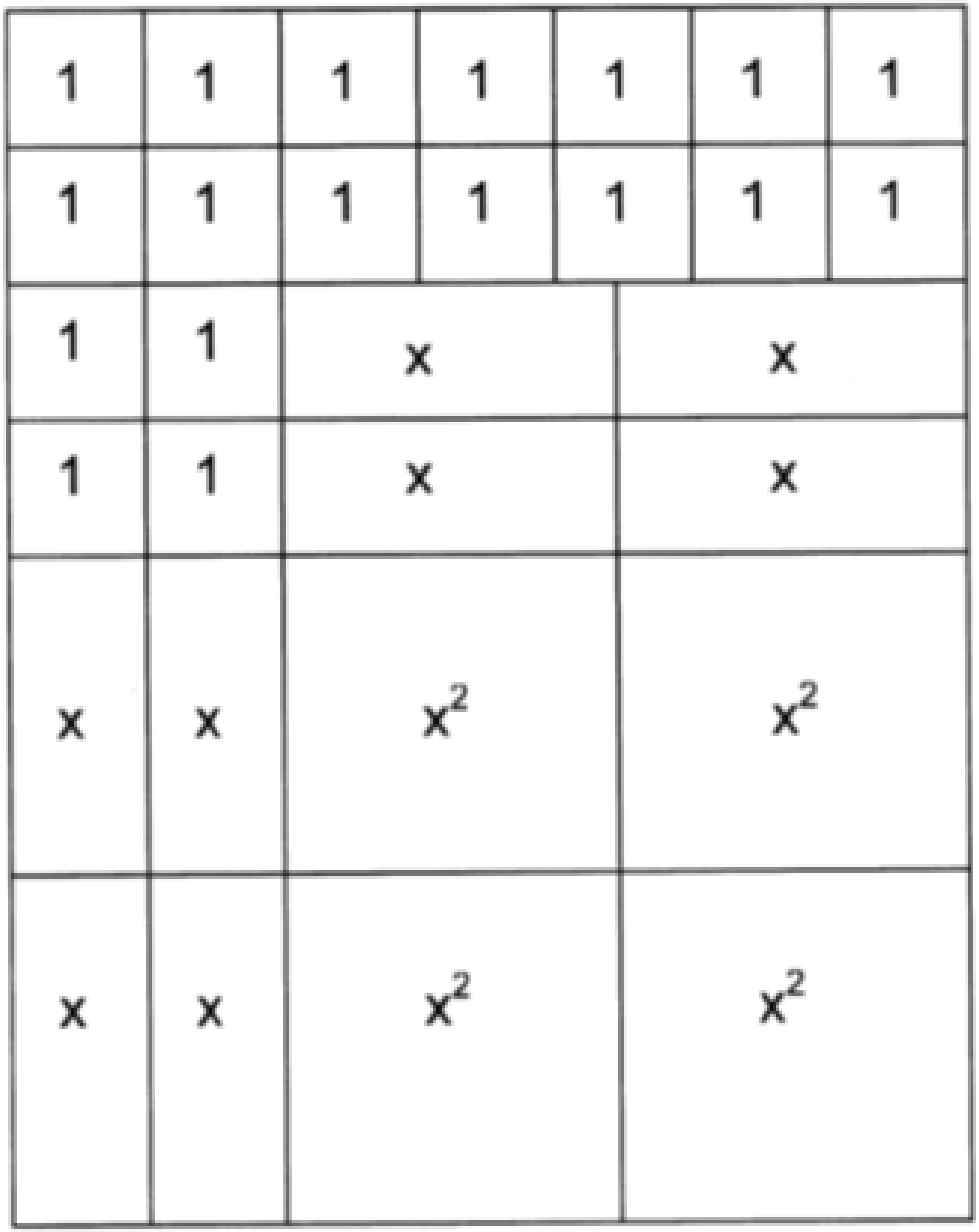
Sample Activities to Get Started

1. Determine the number of different ways that zero (0) can be represented using tiles from a set of 3 blue “one-tiles” and 2 red “one-tiles”.
2. Use the “one-tiles” to model different integer values (e.g. a loss of $4; 2 m above sea level).
3. Create models for integer operations e.g. show that (-4) + (+1) = -3; show that 2(-3) = -6
4. Build an algebra tile model to show that $2x + 3 – 4x – 2 + 5x – 1 = 3x$
5. Build an algebra tile model to show that $(2x + 3) + (-5x –3) = -3x$
6. Build an algebra tile representation of $2(3x + 1)$. Use the model to show that $2(3x + 1) = 6x + 2$.
7. Make two different models of the ratio 3:2.

8. Build algebra tile models for $(x + 1)2$ and $x2 + 1$. Use your models to explain why these expressions are not equivalent.
9. Try to arrange two red “*x*-tiles” and three red “one-tiles” into one rectangular arrangement. (This activity builds understanding of factoring.) Can a rectangular arrangement always be made?
10. Use the red “one-tiles” tiles to show all possible factors of 12.
11. Build a tile train. What is the color and shape of the 200th cube in the train?

Algebra Tile Template




1
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x
x
x
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x^2
x
x
x^2
x^2

Algebra Tile Mat

X

Simon Says

Students need to feel comfortable with algebra tiles in order to use them effectively and learn basic concepts. Introduce the tiles – x2, -x2, x, -x, 1, and -1. Model for students how to "set up" the tiles to show different expressions, as well as defining the basic vocabulary to be used: variable, constant, coefficient, and expression. To ensure that students understand the basics, play "Simon Says." Provide students with sample algebraic expressions and have them use their algebra tiles to show the correct display. Speed up the directions as students become more comfortable using the tiles.

Example:

Simon says show me:

- $2x^2$
- $4x$
- $- x^2$
- 3
- $2x + 3$
- $- x^2 + 4$
- $2x^2 + 6x + 5$
- $-2x^2 - 6x - 5$
- $x^2 - 2x + 3$

You may wish to play a round robin version of "Simon Says" by selecting a student to state an expression with the first person getting it correct providing the next set of directions.

Using Algebra Tiles to Add Like Terms

Use algebra tiles to show each expression and draw your result. Then combine like terms. Show your final answer by circling it.

1.) $3 + x^2$

2.) $x + 3 + 2x$

3.) $3x + 2 + x + 4$

4.) $4x + 1 + 2x + 4$

5.) $4x + 2x - 4 + 5 - x$

6.) $x + 4 + 3x - 3$

7.) $2x^2 + x^2$

8.) $2x^2 + 3x + 2 + 2 x 2$

9.) $x^2 +3 x + 2x2 - x + 2$

Regardless of What You Call the Terms, It's All Just Math!

Translate the following into a mathematical expression.

1. the difference between twice x and y
2. the difference between the square of x and x
3. the quotient of y and 3
4. five times the sum of x and y
5. the sum of 4 times x and y
6. ten less than x
7. the product of a, b, and c
8. the sum of 7 and x
9. x minus 8
10. x less than 8

Try These!

For each of the following, write an expression in terms of the given variable that represents the indicated quantity.

1. The cost of having a mechanic fix your car if he spends *h* hours on the job and charges \$39 for parts and \$45 per hour for labor.

2. The sum of three consecutive even numbers if the first number is *n*.

3. The amount of money in Steve's bank account if he put in *d* dollars the first year, \$600 more the second year than the first year, and twice as much the third year as the second year.

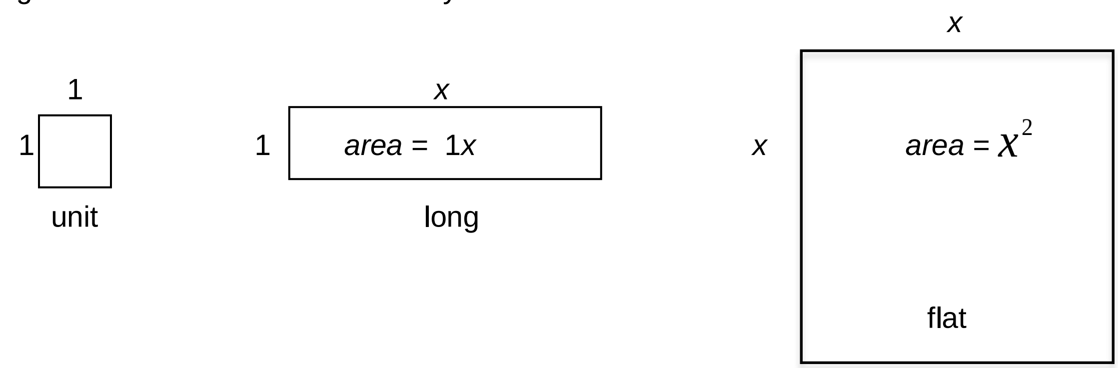
4. The first side of a triangle is *s* yards long. The second side is 3 yards longer than the first side. The third side is three times as long as the second side. What is the perimeter of the triangle in feet?

Introduction to Algebra Tiles

Lesson Overview: To introduce and familiarize students with the use of algebra tiles to multiply two binomials. From the concrete process of algebra tiles, students will discover more representational or abstract methods to solve these types of problems (e.g. the Box Method or FOIL Method).

Introduction to Algebra Tiles

For this exercise you will need three types of algebra tiles, the unit tiles, the rods (or longs), and the flats. Algebra tiles are often referred to by their areas.



To form larger (composite) rectangles using just small squares and rectangles, the tiles must be aligned with their sides of common length, namely, their sides of length 1, put together. Similarly, to form larger rectangles using just large squares and rectangles, the tiles must be aligned with their sides of common length put together. Let’s try it and draw some sketches of the results.

Composite Rectangles Directions

1. Determine whether it is possible to build a composite rectangle with each set of tiles indicated in the following chart. If possible, sketch a composite rectangle you could build. If not, answer “not possible.”

2. For each composite rectangle, label each tile with its area as shown, and then label the length and width of the rectangle.

3. Use what you have done to write the area of each set of tiles as a sum of the areas of the pieces, and then as a product of the length and the width, if possible.

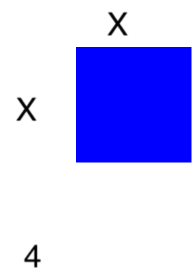
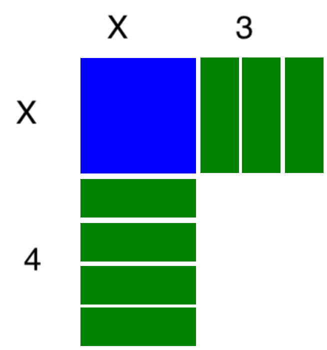
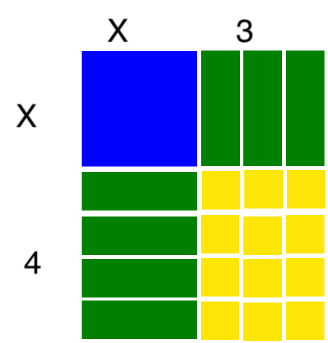
Composite Rectangles

Number of x^2 Tiles	Number of 1x Tiles	Number of 1 Tiles	Is a composite rectangle possible?	Sketch	Algebraic Expression for the Area as a Sum Algebraic Expression for the Area as a Product
1	3	2			
1	4	4			
2	3	1			
0	3	6			

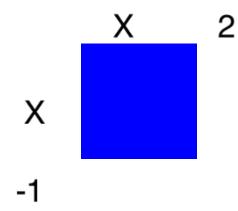
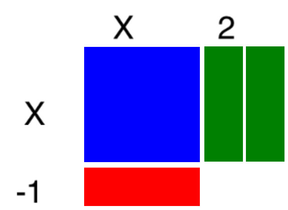
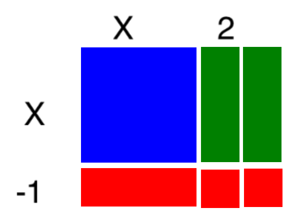
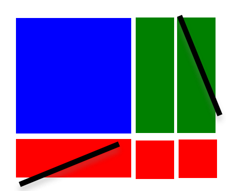
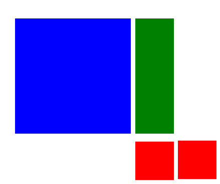
Composite Rectangles

Number of x^2 Tiles	Number of 1x Tiles	Number of 1 Tiles	Is a composite rectangle possible?	Sketch	Algebraic Expression for the Area as a Sum / Algebraic Expression for the Area as a Product
1	6	4			
1	4	0			
2	7	6			
1	7	10			

A Step-By-Step Process: Product of Two Binomials

Step	Directions	Work
1	$(x + 3)$ $(x + 4)$ Write x and 3 horizontally at the top and x and 4 vertically on the far left.	X 3 X 4
2	Students start with blue tile(s) (tiles with area x^2) placed at the top left.	
3	Students place the appropriate number of green tiles (tiles with area x).	
4	Students fill in the empty space with the appropriate number of yellow tiles (tiles with area of 1 unit).	
5	Count the number of each type of tile.	x^2 tiles = 1 x tiles = 7 1 (unit) tiles = 12
6	Write your expression.	$x^2 + 7x + 12$

A Step-By-Step Process: Product of Two Binomials

Step	Directions	Work
1	(x + 2) (x - 1) Write x and 2 horizontally at the top and x and -1 vertically on the far left. . (Note: We use –1 to represent the minus 1 since x – 1 is the same as x + (-1).)	X 2 X -1
2	Students start with blue tile(s) placed at the top left.	
3	Students place the appropriate number of green or red tiles. (Note: Red tiles represent negative values.)	
4	Students fill in the empty space with the appropriate number of red/green unit tiles. (Note: Use red tiles if the signs are opposite, use green tiles if the sign are the same.)	
5	Tiles with the same area but opposite colors will cancel one another out.	
6	Count the number of each type remaining tiles.	x^2 tiles = 1 x tiles = 1 1 (unit) tiles = -2 
7	Write your expression.	$x^2 + x - 2$

Practice with Binomials

Sketch the algebra tiles that would be used to represent the product of the following binomials.

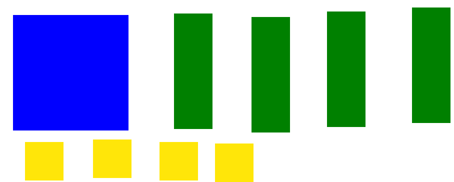
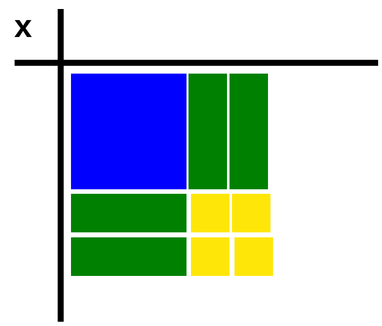
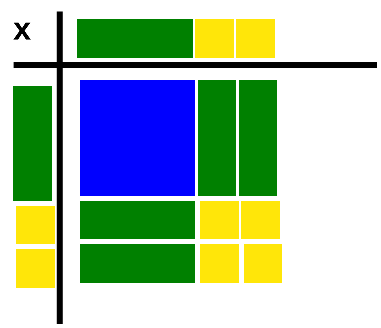
1. $(x + 1)(x + 2)$

2. $(x + 2)(x - 3)$

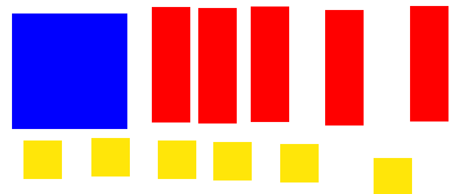
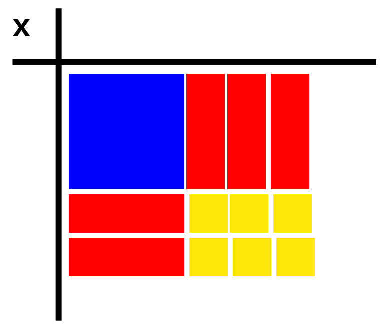
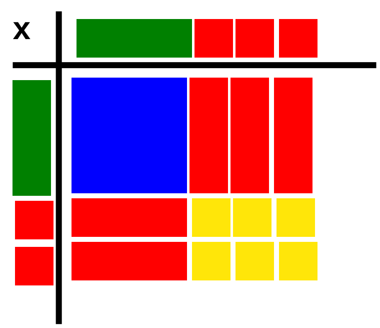
3. $(x - 1)(x + 3)$

4. $(x - 2)(x - 3)$

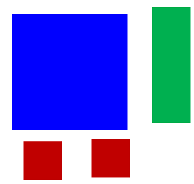
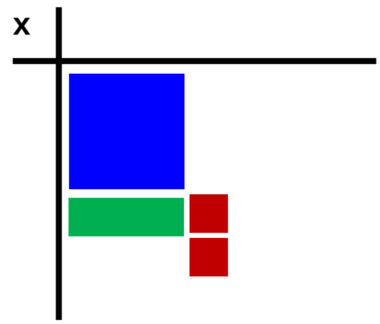
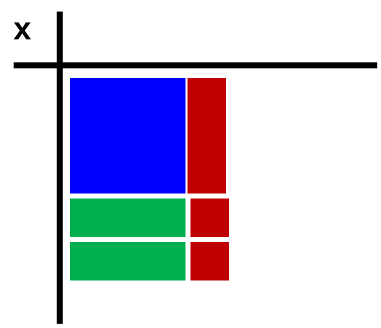
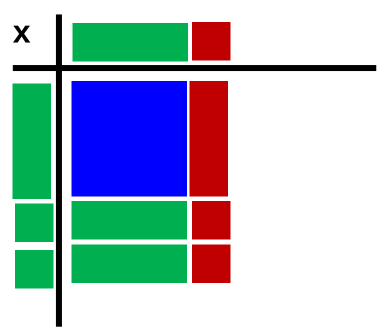
A Step-By-Step Process: Factoring

Step	Directions	Work
1	$x^2 + 2x + 4$ Identify tiles and lay them out.	
2	Put tiles into a rectangular array.	
3	Demonstrate how to determine the factors of the trinomial by placing the correct length tile on the vertical and horizontal axis. Identify the factors: x + 2 and x + 2	
4	Write your expression.	$(x + 2)\ (x + 2) = x^2 + 2x + 4$

A Step-By-Step Process: Factoring with Negative Trinomials

Step	Directions	Work
1	$x^2 - 5x + 6$ Identify tiles and lay them out.	
2	Put tiles into a rectangular array. Make sure the tiles are shown using positive tiles and negatives correctly.	
3	Demonstrate how to determine the factors of the trinomial by placing the correct length tile on the vertical and horizontal axis. Discuss that unlike tiles result in negative or red tiles, whereas like signs result in positive tiles. Identify the factors: x -2 and x - 3	
4	Write your expression.	$(x - 2)\ (x - 3) = x^2 -5x + 6$

A Step-By-Step Process: Factoring with Negative Trinomials and Zero Pairs

Step	Directions	Work
1	x^2 + x - 2 Identify tiles and lay them out.	
2	Put tiles into a rectangular array. Make sure the tiles are shown using positive tiles and negatives correctly. Ask: how can we complete a recangle? What can be added to an expression that doesn't change the expression? Students should answer: Zero pairs.	
3	Add a zero pair of x tiles (one red/one green) and discuss how it does not change the trinomial's value.	
3	Demonstrate how to determine the factors of the trinomial by placing the correct length tile on the vertical and horizontal axis. Discuss that unlike tiles result in negative or red tiles, whereas like signs result in positive tiles. Identify the factors: x -1 and x + 2	
4	Write your expression.	$(x - 1)\ (x + 2) = x^2 -5x + 6$

Practice with Factoring

1. Draw a picture representing $x^2 + 3x + 2$.

2. Draw a picture representing the expression from #1 as a rectangle.

3. What are the factors of $x^2 + 3x + 2$?

4. Factor:

   $x^2 - 5x + 4$

   $x^2 + 7x + 12$

   $x^2 - 9x + 20$

   $x^2 + x – 6$

   $x^2 – x – 12$

   $x^2 – 3x – 10$

A Few Websites to Get You Started!

Online Resources for Teaching and Strengthening Fundamental, Quantitative, Mathematical, and Statistical Skills. NICHE. A wide array of resources for the different types of mathematical skills. **http://serc.carleton.edu/NICHE/teaching_materials_qr.html#partone**

National Library of Virtual Manipulatives for Math - All types of virtual manipulatives for use in the classroom from algebra tiles to fraction strips. This is a great site for students who need to see the "why" of math. http://nlvm.usu.edu/en/nav/index.html

TES. With more than 2.3 million registered online users in over 270 countries and territories, TES provides a wealth of free resources in all academic areas. http://www.tes.co.uk/

Algebra 4 All. A website from Michigan Virtual University with an interactive site for using algebra tiles to solve various types of problems.
http://a4a.learnport.org/page/algebra-tiles

Working with Algebra Tiles. An online workshop that provides the basics of using algebra tiles in the classroom.
http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles.htm

Teaching Algebra Using Algebra Tiles. An instructor site that provides information on teaching algebra, as well as basic algebraic concepts.
http://www.jamesrahn.com/homepages/algebra_tiles.htm

Key Elements to Algebra Success 46 lessons, homework assignments, and videos.
http://ntnmath.keasmath.com/