

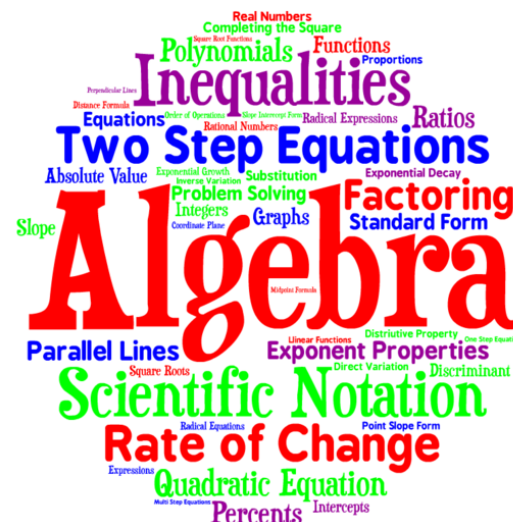
# From Integers to Factoring – Using Algebra Tiles in the Classroom

2019 GED Annual Conference



# During this Workshop, we will...

- Practice using algebra tiles to help students obtain conceptual understanding of the fundamentals of algebra
  - Positive and negative integers
  - Additive inverse (Zero Pairs)
  - Binomials, polynomials
- Discuss importance of contextualizing instruction to help students see the real-world application of algebra
- Identify and use a variety of online and other instructional resources for the classroom.



# What Are Some Ways to Solve These Problems

- Each section of the Bradley Center holds 258 people. There are 72 sections. If all of the seats are filled for a Buck's game, how many people would there be?
- How many bows can you make from  $3\frac{2}{3}$  meters of ribbon if you need  $\frac{2}{5}$  meters of ribbon to make each bow?



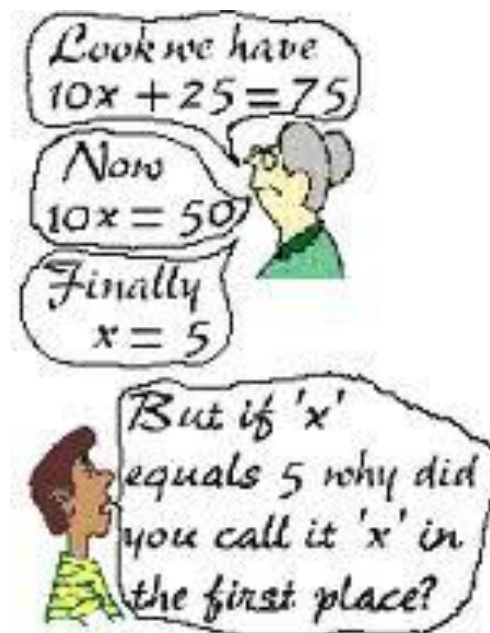
# Would the Same Methods Work?

- Each section of the Bradley Center holds  $p$  people. There are  $s$  sections. If all of the seats are filled for a Buck's game, how many people would there be?
- How many bows can you make from  $r$  meters of ribbon if you need  $m$  meters of ribbon to make each bow?

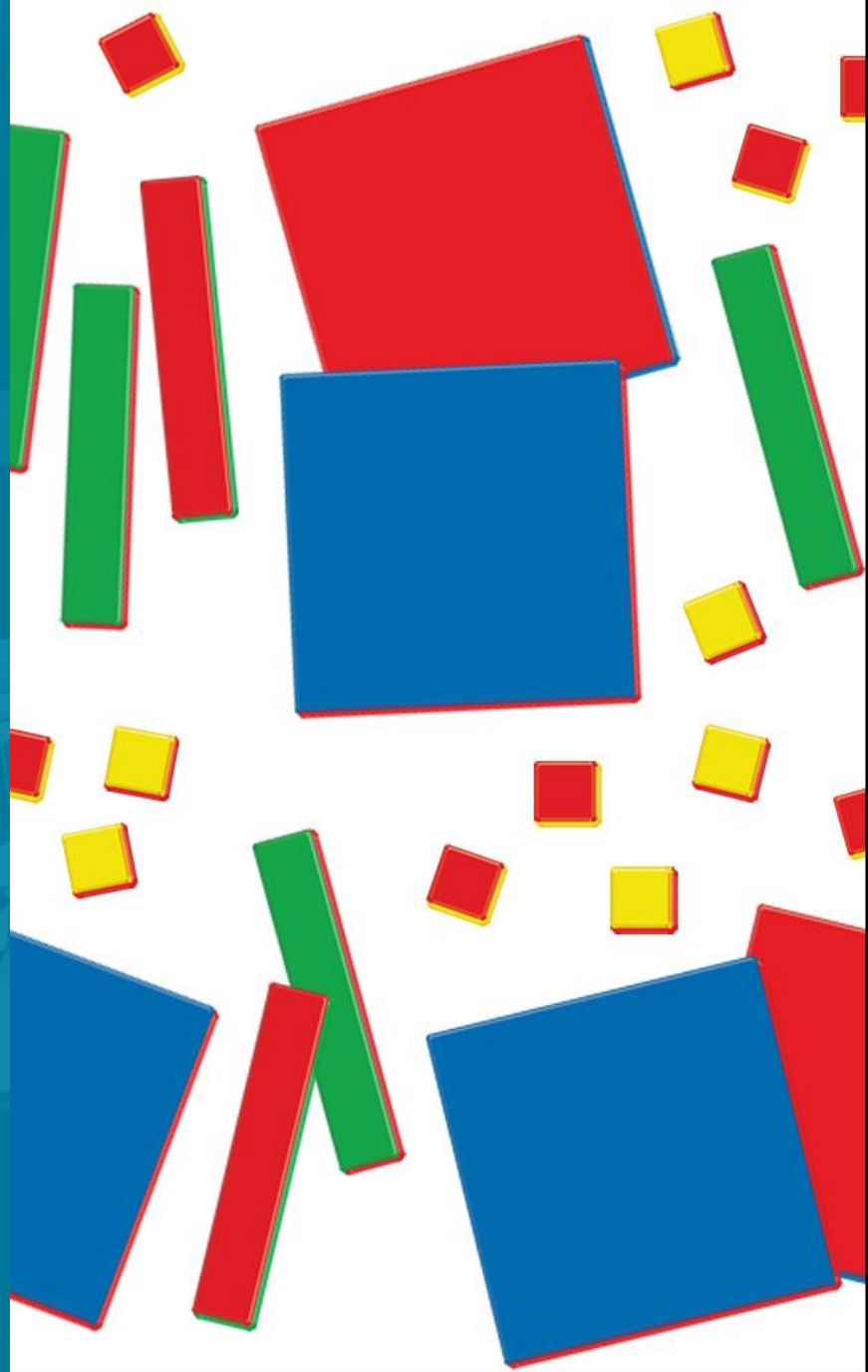


# Remember . . .

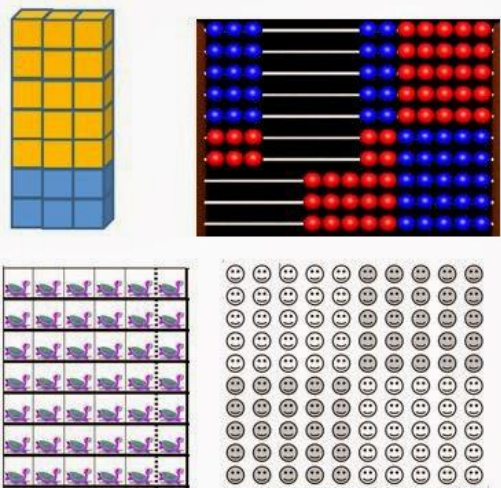
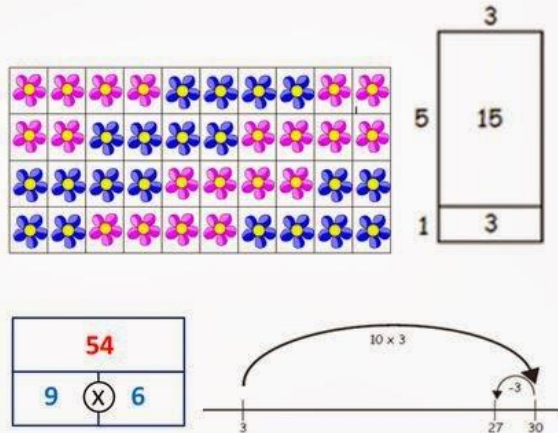
- Arithmetic is doing something to numbers to get an answer.
- Algebra is exploring the relationships between numbers.



# Helping Students Build Understanding of Basic Algebraic Concepts



# C-R-A – Essential for Understanding

Concrete	Representational	Abstract															
Students manipulate hands-on, concrete materials	Students draw and observe diagrams, or watch the teacher touching and moving hands-on materials	Numbers and mathematical symbols															
		<table border="1" data-bbox="1344 714 1760 843"><tr><th colspan="5">x 4 Patterns</th></tr><tr><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td></tr><tr><td>24</td><td>28</td><td>32</td><td>36</td><td>40</td></tr></table> <div data-bbox="1327 891 1767 1112"><div><math>8 \times 5</math> <math>(4 \times 2) \times 5</math> <math>4 \times (2 \times 5)</math> <math>4 \times 10</math> <math>40</math></div><div><math>45 \div 5</math> <math>(50-5) \div 5</math> <math>(50 \div 5) - (5 \div 5)</math> <math>10-1</math> <math>9</math></div></div>	x 4 Patterns					4	8	12	16	20	24	28	32	36	40
x 4 Patterns																	
4	8	12	16	20													
24	28	32	36	40													

# Manipulatives for Algebra (CRA)

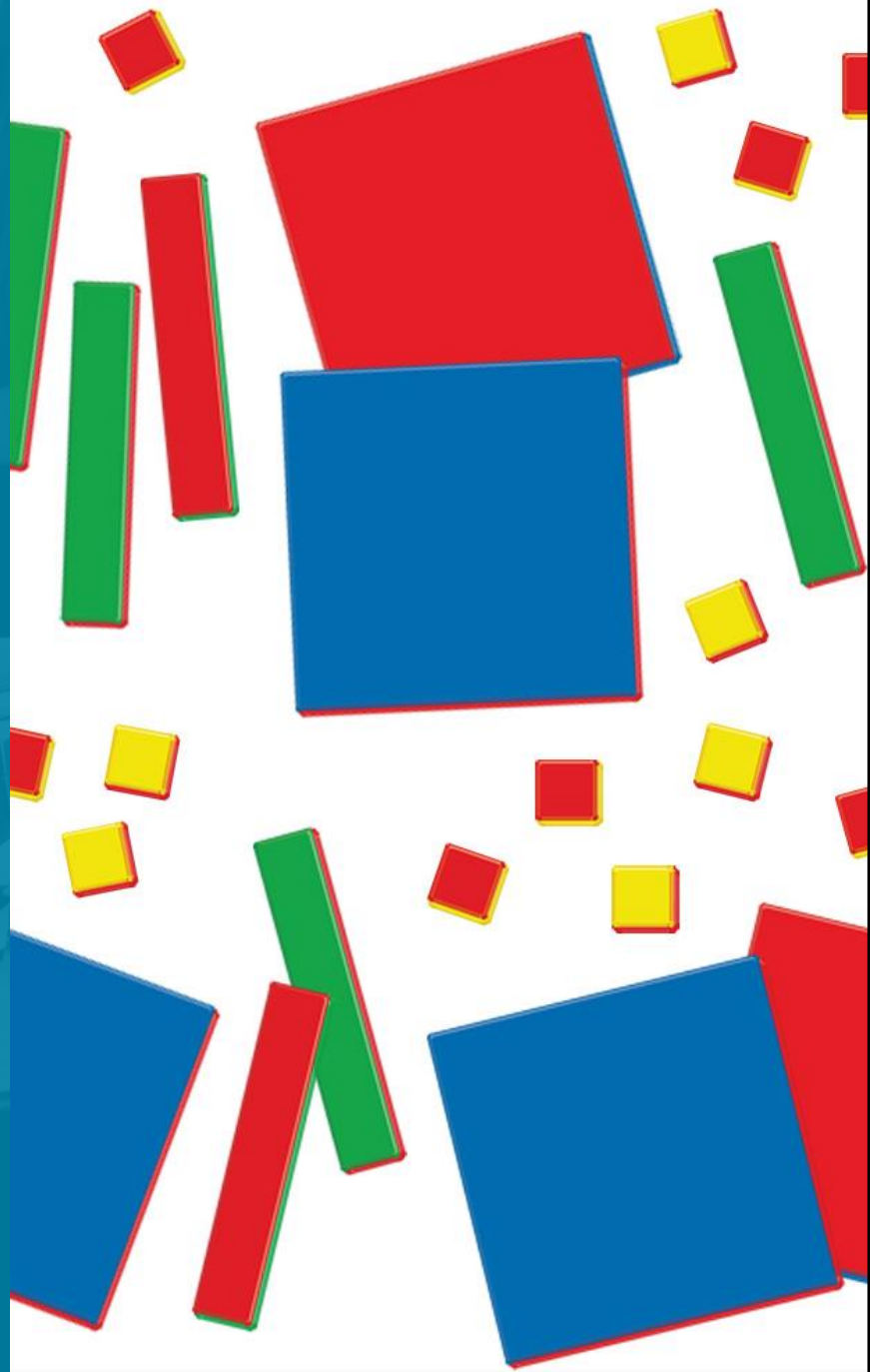


**algebra tiles, algetiles, math tiles, virtual tiles**

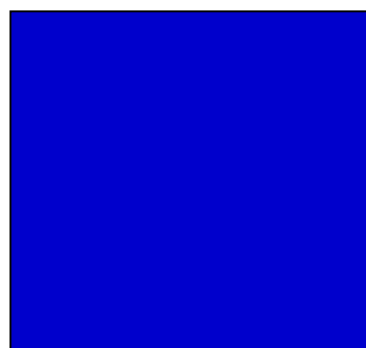
- Students with access to virtual manipulatives achieved higher gains than those students taught without manipulatives.
- Students using hands-on and manipulatives were able to explain the how and why of algebraic problem solving.



# Holding Algebra in Your Hands



# Introduction to Algebra Tiles



$x^2$



$x$



$1$

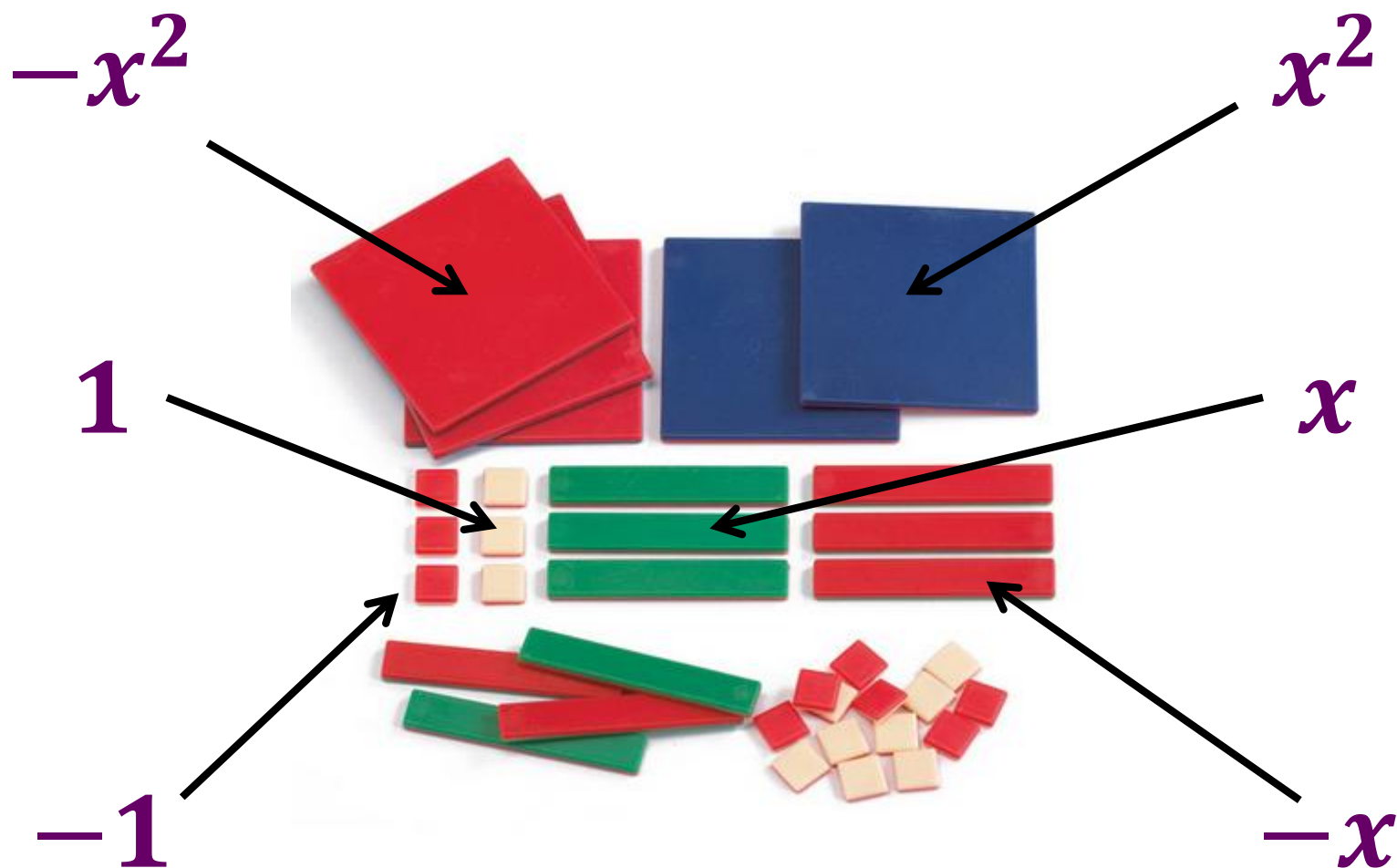
**Positive Tiles**



**Negative Tiles**

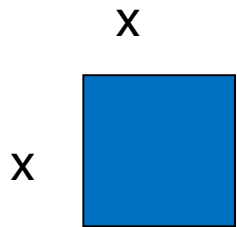
**Remember, they could be called  $x$ ,  $y$ ,  $b$ ,  $t$ , etc.**

# Introduction to Algebra Tiles



# Introduction to Algebra Tiles

Each tile represents an area.



Area of large square =  $x(x) = x^2$



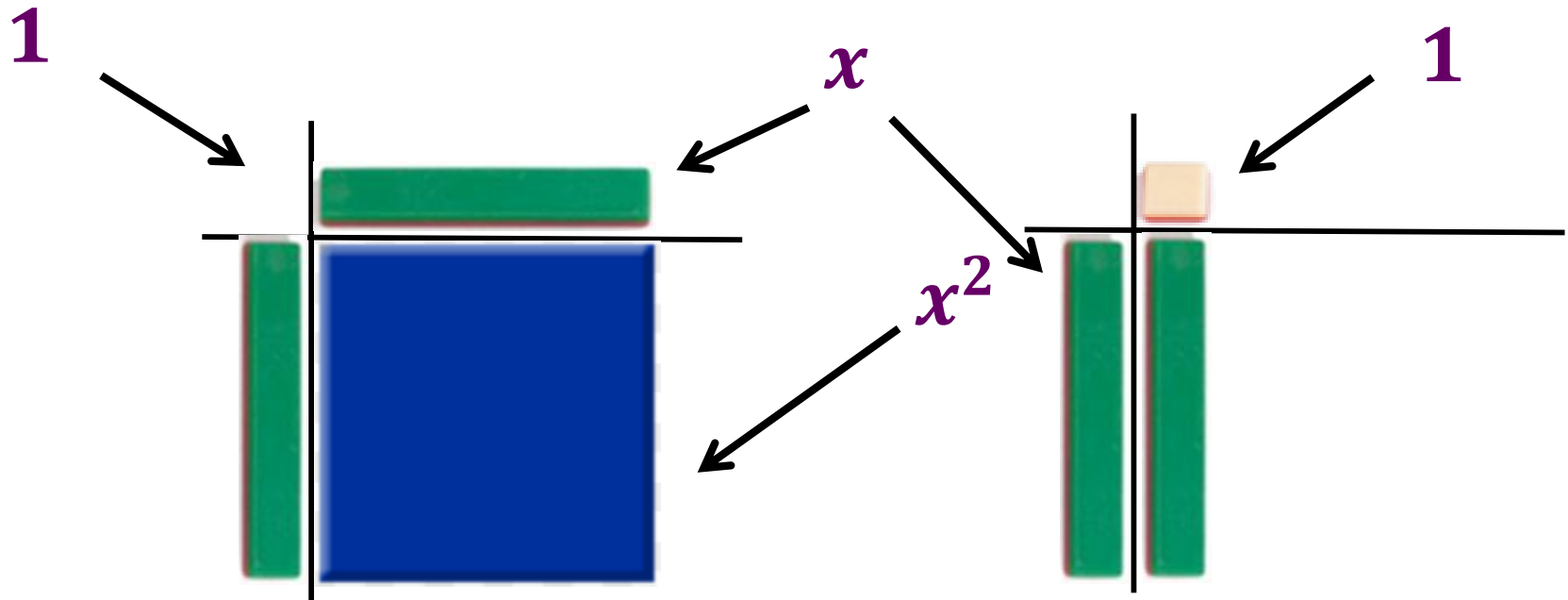
Area of rectangle =  $1(x) = x$



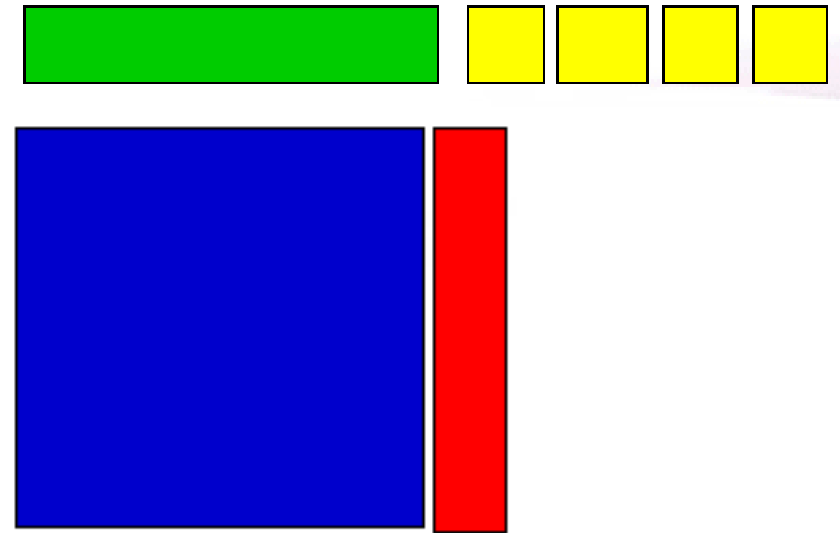
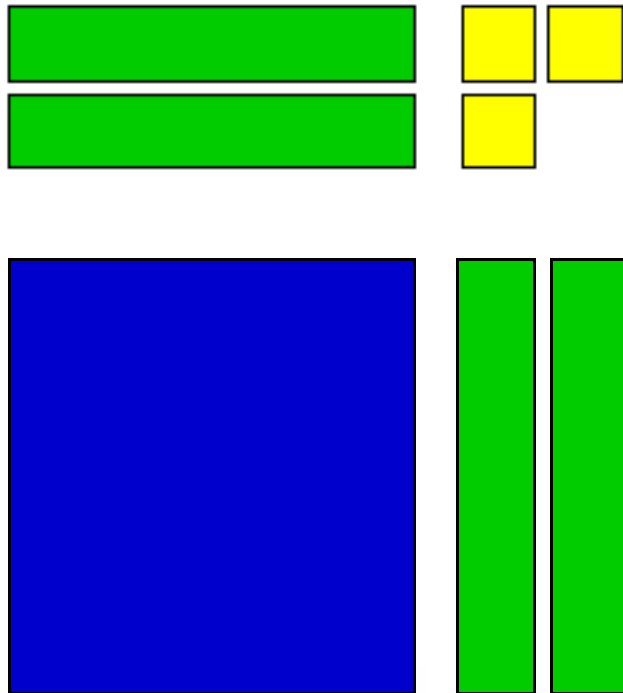
Area of small square =  $1(1) = 1$

*Note: Tiles are not to scale. 10 little tiles don't equal 1 big tile!*

# Look at the Relationships among the Tiles



# What's My Polynomial?



# Simon says show me . . .

- $2x^2$
- $-x^2$
- 3
- $2x + 3$
- $2x^2 + 6x + 5$
- $-2x^2 - 6x - 5$



# Additive Inverse (Zero Pairs)

What you add to a number to get zero.  
The negative of a number.

Example:

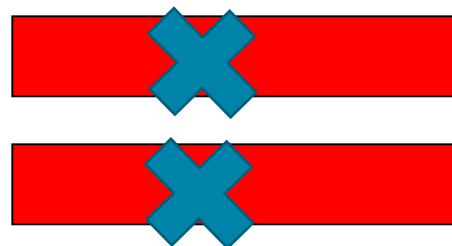
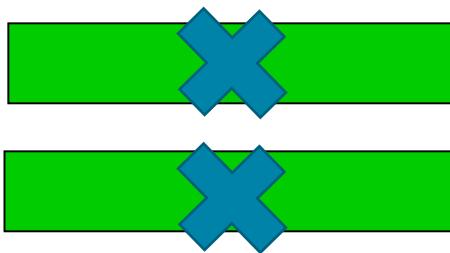
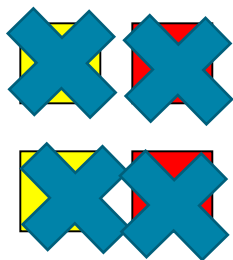
The additive inverse of  $-5$  is  $+5$ , because  $-5 + 5 = 0$

The additive inverse of  $+5$  is  $-5$ , because  $+5 - 5 = 0$

$$\begin{array}{c} -5 \\ \text{Number} \end{array} + \begin{array}{c} 5 \\ \text{Additive Inverse} \end{array} = 0$$

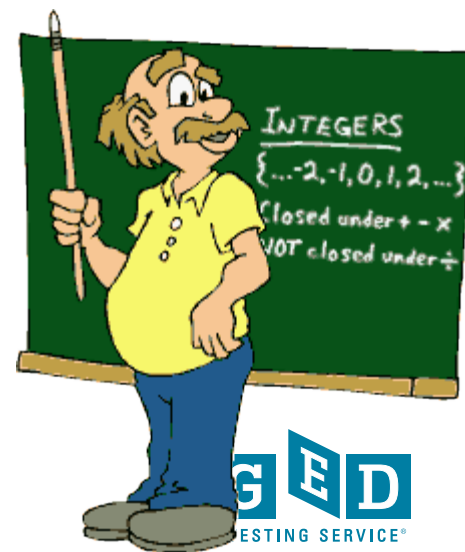
$$\begin{array}{c} 14 \\ \text{Number} \end{array} + \begin{array}{c} -14 \\ \text{Additive Inverse} \end{array} = 0$$

## Algebra Tiles – Additive Inverse



# Use Algebra Tiles to Model Integer Addition

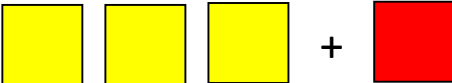
- Addition is “combining.”
- Combining involves the forming and removing of zero pairs.
- Remember, an integer is a number with no fractional part.




# Addition of Integers

$$(+3) + (+1) =$$
The diagram shows three yellow squares, followed by a plus sign, and then one yellow square, representing the addition of positive integers.

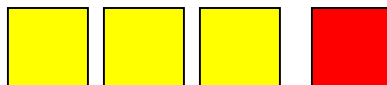
$$(-2) + (-1) =$$
The diagram shows two red squares, followed by a plus sign, and then one red square, representing the addition of negative integers.

$$(+3) + (-1) =$$
The diagram shows three yellow squares, followed by a plus sign, and then one red square, representing the addition of a positive integer and a negative integer.

$$(+4) + (-4) =$$
The diagram shows four yellow squares, followed by a plus sign, and then four red squares, representing the addition of a positive integer and its opposite negative integer.

# Addition of Integers

$$(+3) + (-1) =$$



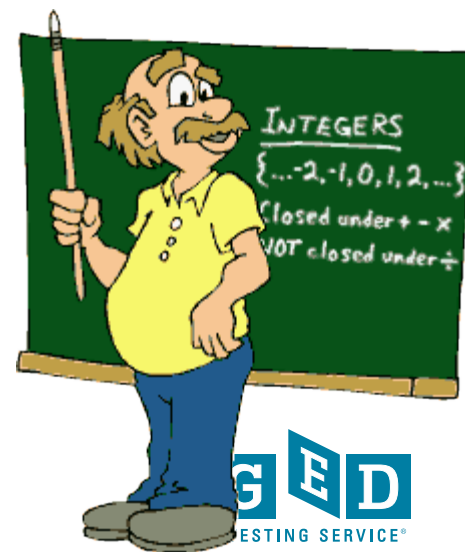
$$(+4) + (-4) =$$




Don't forget that a positive and a negative "cancel" each other out!

# Use Algebra Tiles to Model Integer Subtraction

- Subtraction can be interpreted as “take-away.”
- Subtraction can also be thought of as “adding the opposite.”



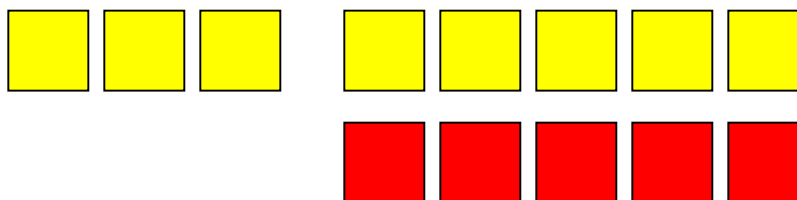
# Subtraction of Integers

$$(+5) - (+2) =$$
Five yellow squares are arranged horizontally, representing the positive value 5.

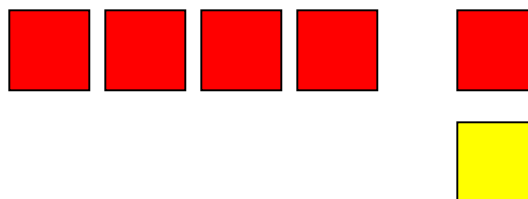
$$(-4) - (-3) =$$
Four red squares are arranged horizontally, representing the negative value -4.

# Subtracting Integers – It's Your Turn!

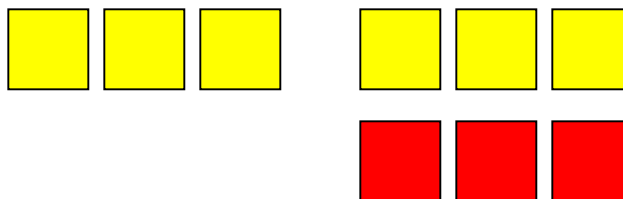
$$(+3) - (-5)$$



$$(-4) - (+1)$$



$$(+3) - (-3)$$



# Why is Understanding Additive Inverse Important?

- The additive inverse is important for understanding how to cancel terms when solving for variables in equations and formulas.
- When a number is added to its **additive inverse**, the result is zero. This is useful for getting rid of terms.

*This important concept ensures that students understand the underlying concept of working with integers. Often, students rely on the old adage, “if you have two negatives change it to a positive.” They have a “process,” but not an understanding of how that process works.*

# Combining Polynomials

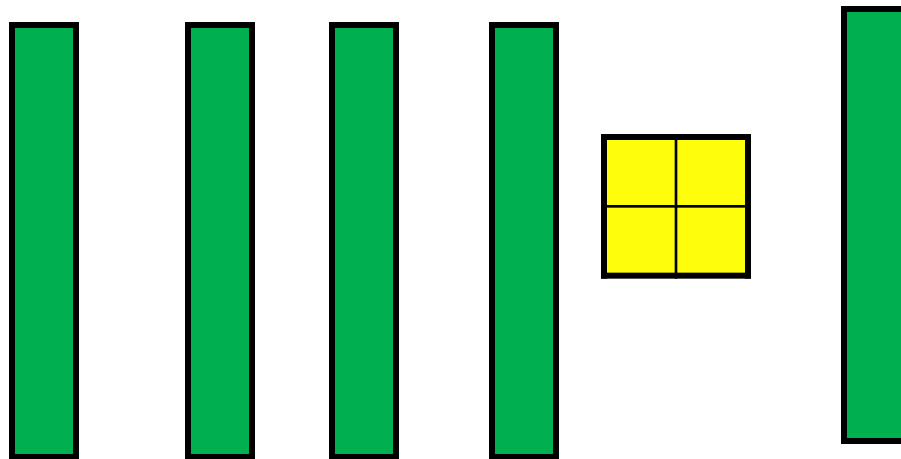
Terms in an expression are **like terms** if they have identical variable parts.

You can **combine terms** that are alike.

You **cannot combine** terms that are unlike.

# Combining Like Terms

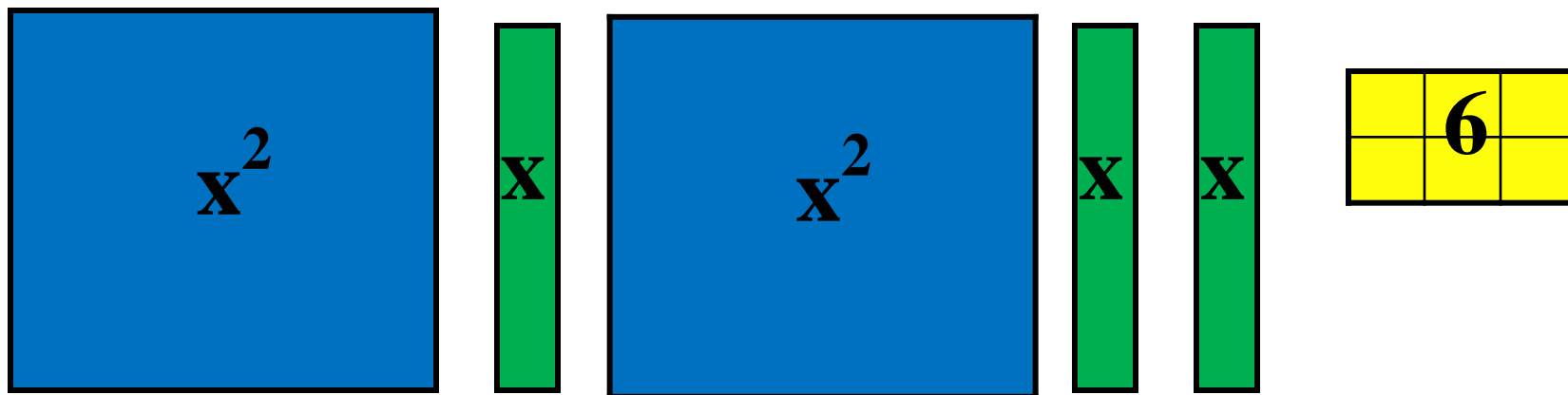
How much  
do I have  
here?



I have  $5x + 4$

# Combining Like Terms

You can only combine terms that are alike, or terms that have the same variable.



Answer:  $2x^2 + 3x + 6$

# Show Me! Now Simplify!

1.  $x + 3 + 2x =$

$$3x + 3$$

2.  $3x + 2 + x + 4 =$

$$4x + 6$$

3.  $2x^2 + 2x + 3 + x^2 + 1 =$

$$3x^2 + 2x + 4$$

4.  $-4 + 2x + 3x^2 - x - 3x =$

$$3x^2 - 2x - 4$$

# Expressions and Equations

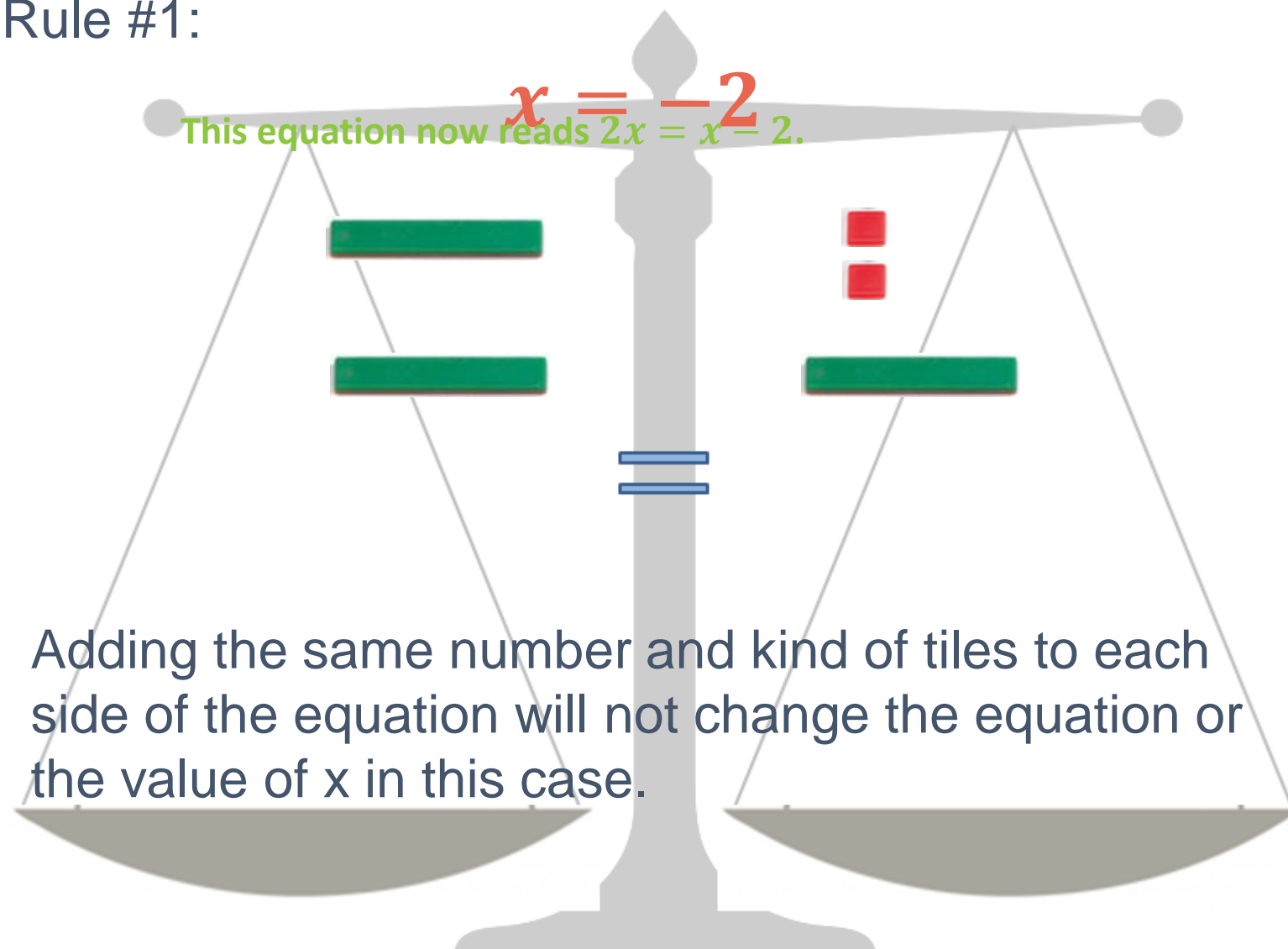
## Basic Rules

1. Both sides of the equation must remain balanced.
2. Any pair of opposite signs/charges (or zero pair) cancel out, has no value, and must be taken out of the mat.



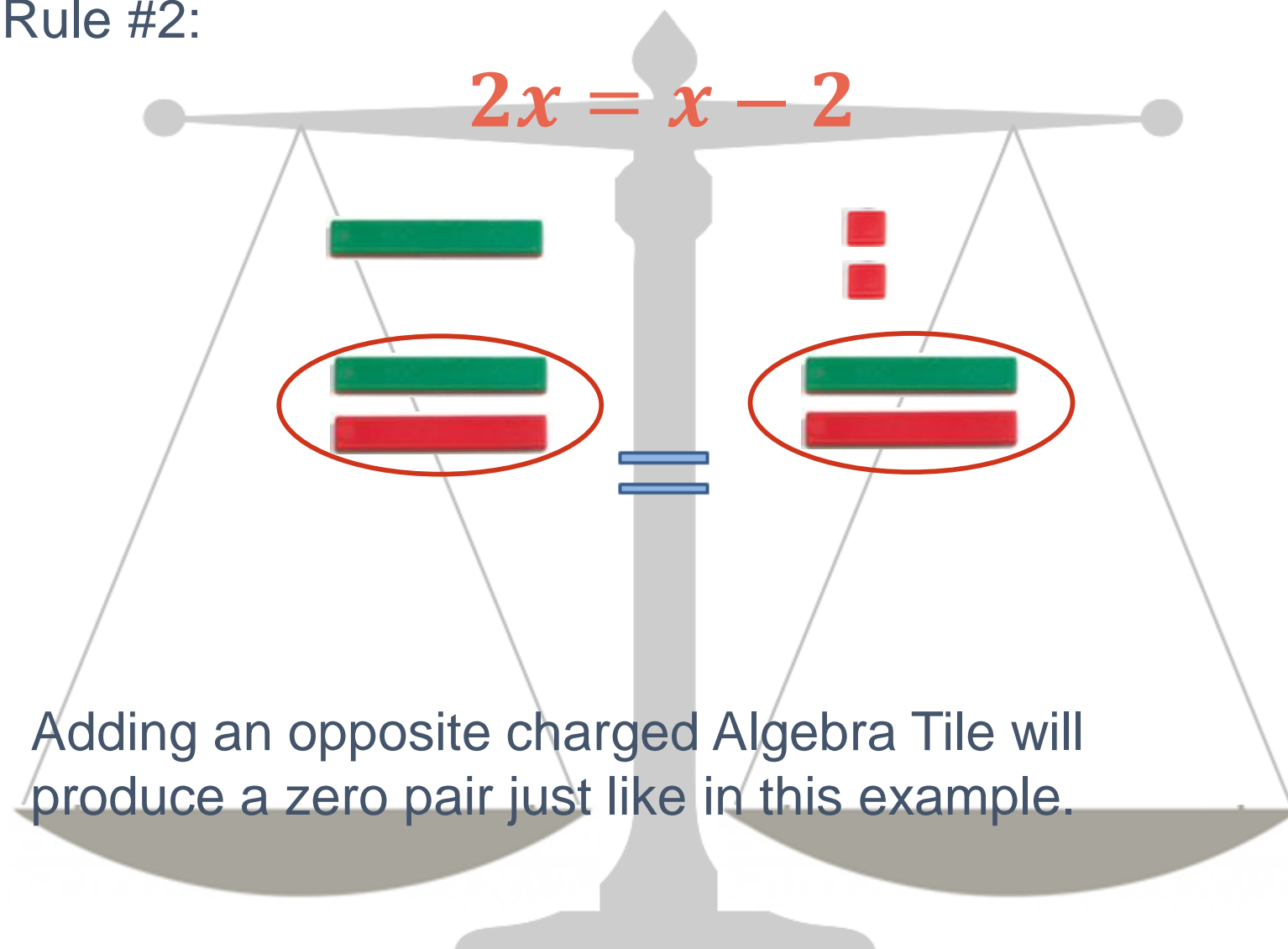
# Modeling Equations

Rule #1:



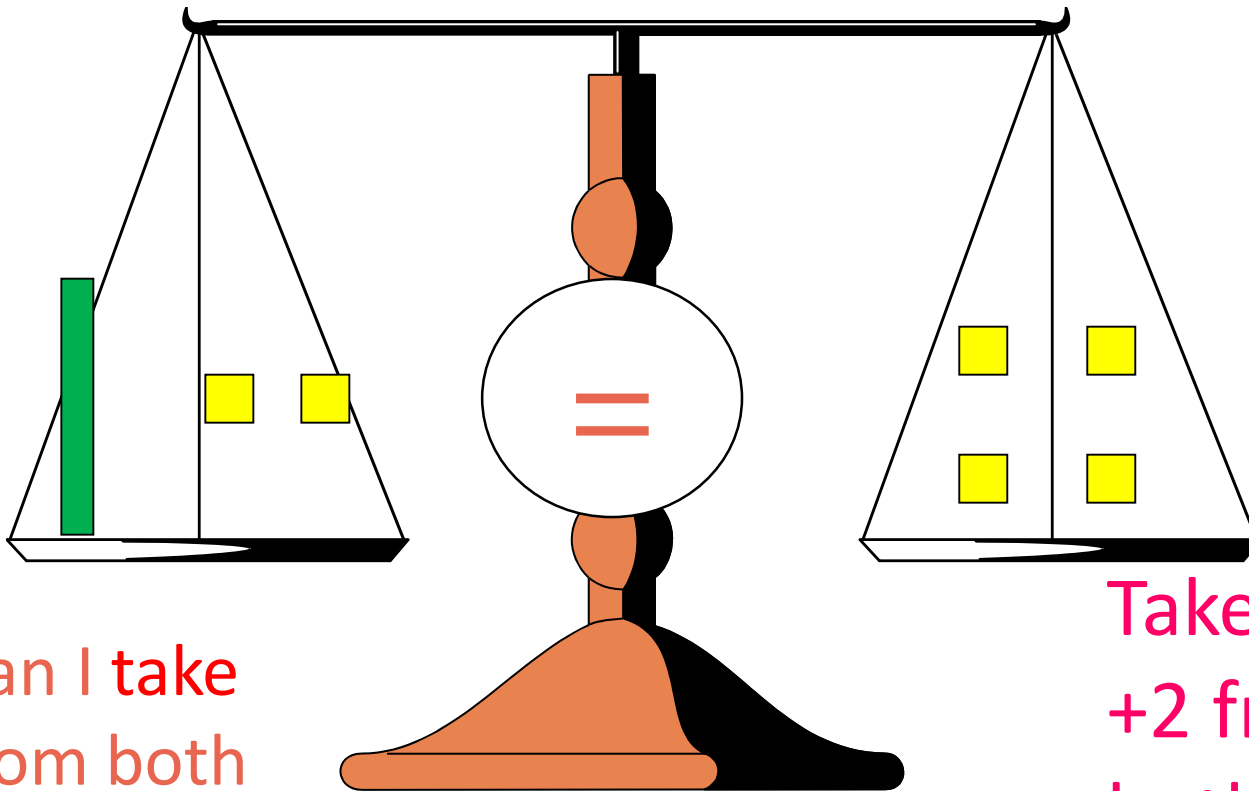
# Modeling Equations

Rule #2:



Adding an opposite charged Algebra Tile will produce a zero pair just like in this example.

# Use Algebra Tiles to Find the Value of $x$

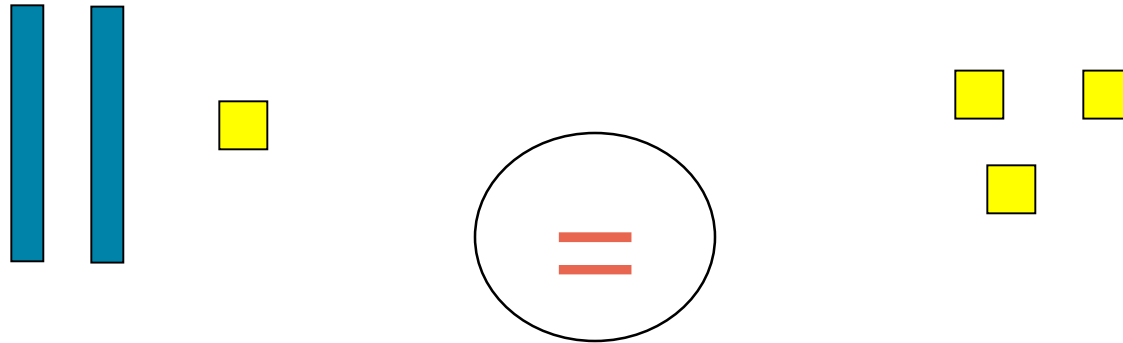


What can I take  
away from both  
sides?

$$x = 2$$

Take away  
+2 from  
both sides.

# Solve for $2x + 1 = 3$



What can I take away from both sides?

If  $2x = 2$ , how do I solve?

$$x = 1$$

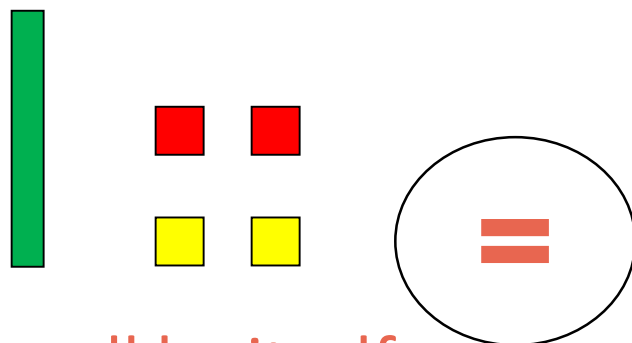
Take away  $+1$  from both sides.

Divide both sides in half.

# Solve for $x - 2 = 3$

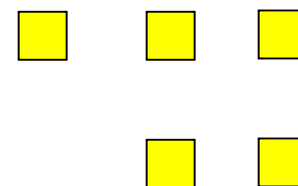
They make  
0 so take  
them away.

What happens to -  
 $2 + (+2)$



I want to get  $x$  all by itself.

How do I get rid of  $-2$  ?



Add  $+2$  to both  
sides.

$$x = 5$$

# Let's Collect Tiles!

## The Rules!

- Big squares can't touch little squares.
- Little squares should all be together.
- Tiles should always be in a rectangular array.

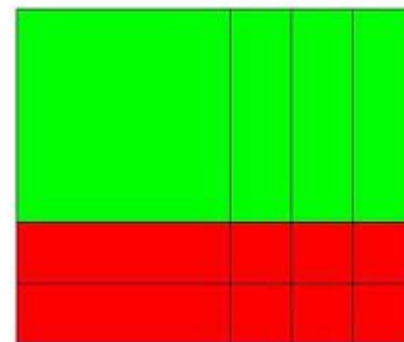
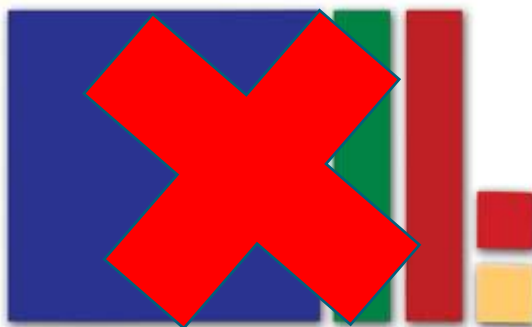
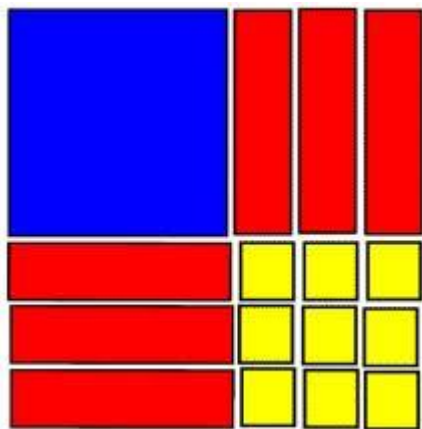
$$2x^2 + 7x + 6$$

Which looks best?



# Let's Play

How many rectangular arrays do you see?



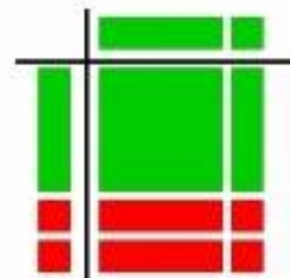
# Algebra Tiles – Time to Collect Tiles!

- $x^2 + 6x + 8$
- $x^2 - 4x + 3$
- $x^2 + 7x + 6$

# Multiplying Polynomials

It's just like figuring area!

- Place one term at the top of the grid
- Place the second term on the side of the grid
- Maintain straight lines when filling in the grid
- The inner grid is your answer!



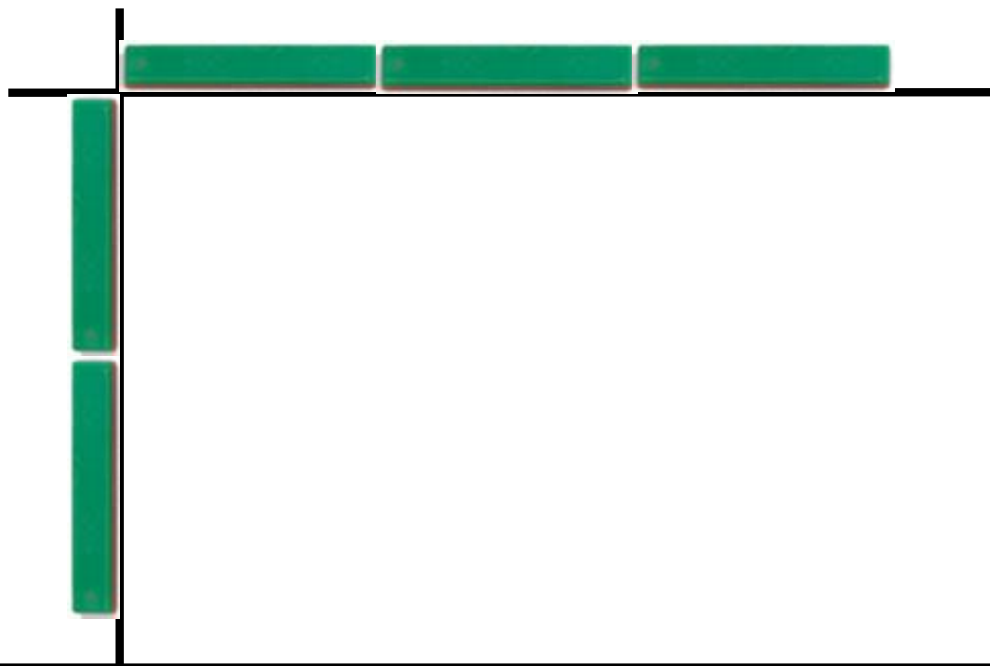
# Let's Multiply Polynomial

$$(3x)(2x) = ?$$

Step 1: Gather all the tiles you need for each term.

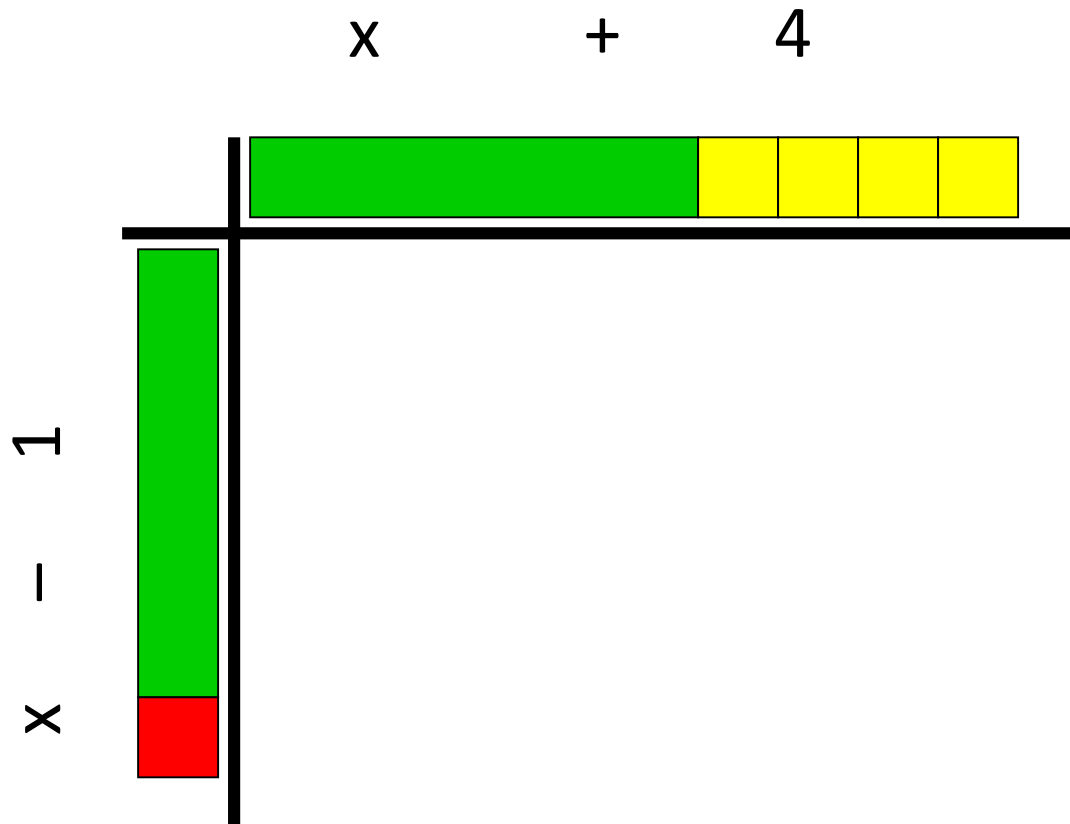


Step 2: Lay out the tiles on opposite axes of the product mat just as shown below.



# Multiplying Polynomials

$$(x - 1)(x + 4) = x^2 + 3x - 4$$



# Multiplying Polynomials

- $(x + 2)(x + 1) =$
- $(x + 5)(x + 3) =$
- $(2x + 2)(2x + 1) =$

Binomials are  
FUN!



**Your Turn!**

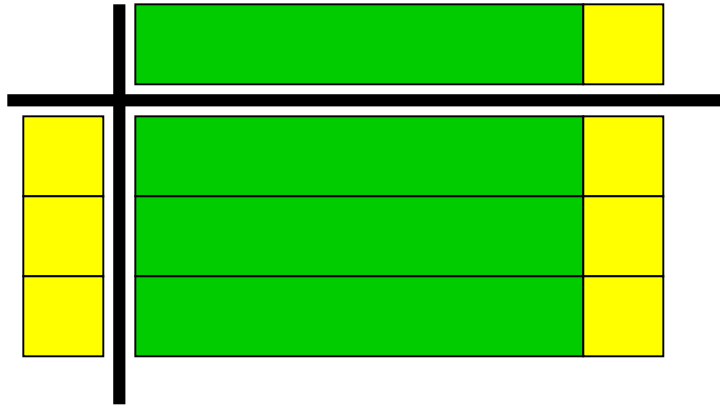
# Factoring Polynomials

- Algebra tiles can be used to factor polynomials. Use tiles and the frame to represent the problem.
- Use the tiles to fill in the array so as to form a rectangle inside the frame.
- Be prepared to use zero pairs (when needed) to fill in the array.
- Solve!

# Factoring Polynomials

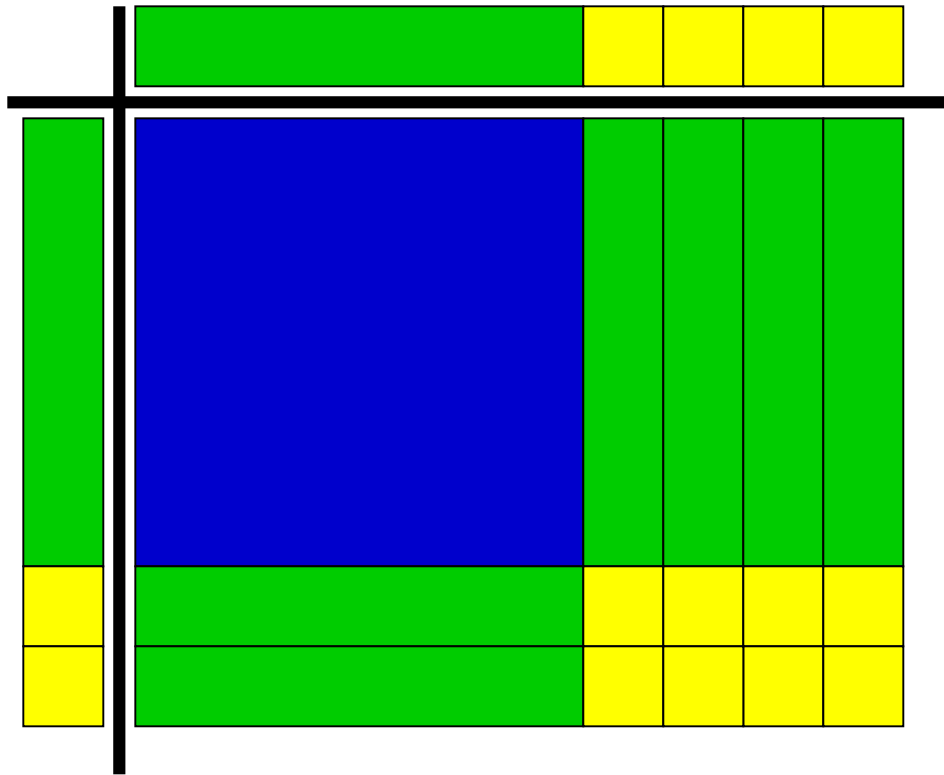
$$3x + 3$$

$$3(x+1)$$



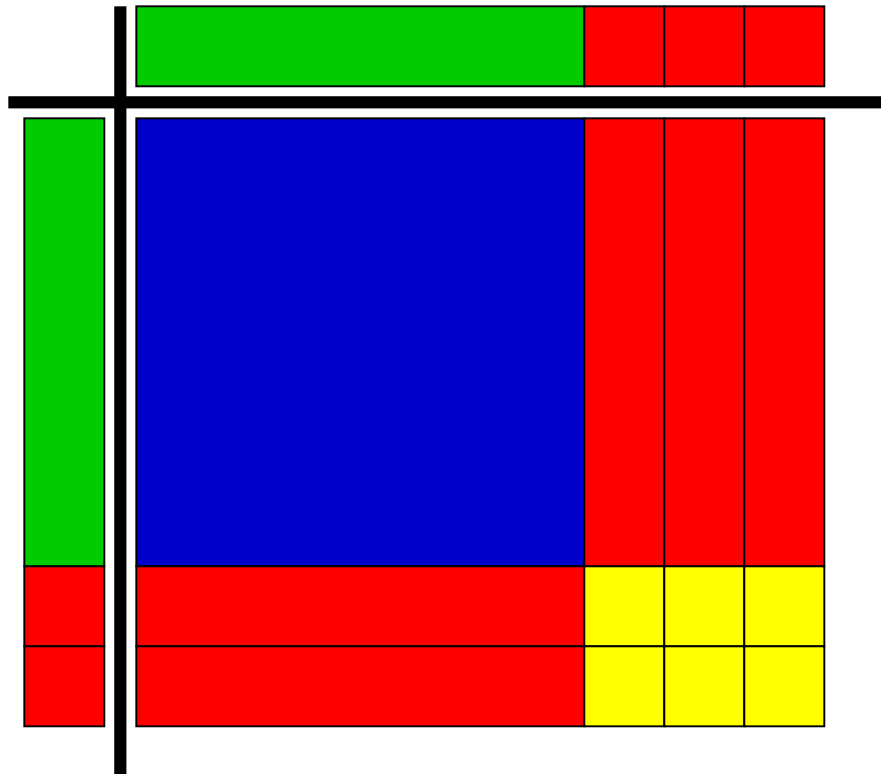
# Factoring Polynomials

$$x^2 + 6x + 8 = (x + 4)(x + 2)$$



# Factoring Polynomials

$$x^2 - 5x + 6 = (x - 3)(x - 2)$$



# Time to Gather Up the Tiles!



# Additional Practice Activities

Factoring Polynomials Using Algebra Tiles - Del Mar College

<HTTP://DMC122011.DELMAR.EDU/MATH/MLC/QEPMATHSEMINARS/FACTORINGTRINOMIALSALGEBRATILESSTUDENTACTIVITY.PDF>

Multiplying Polynomials Using Algebra Tiles – Virginia Dept. of Education

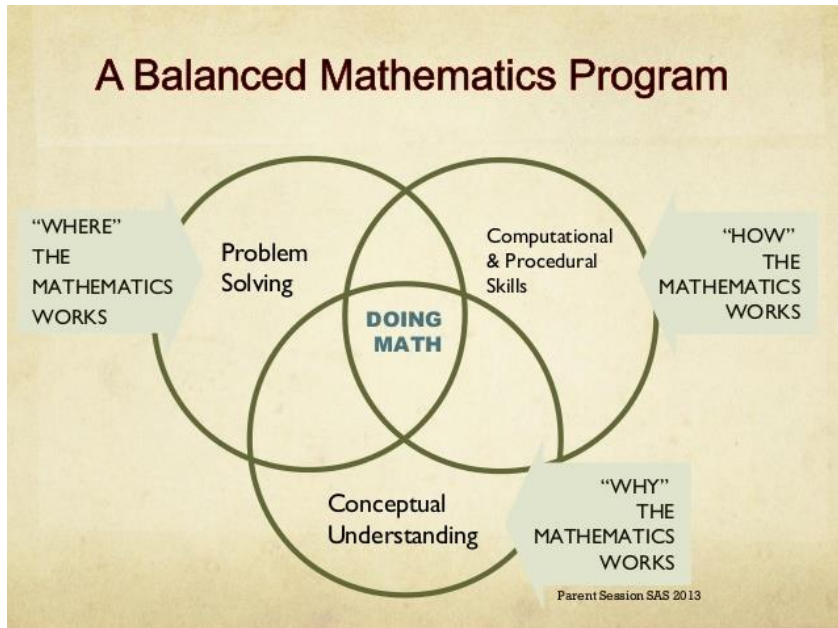
Mathematics Enhanced Scope and Sequence – Algebra I

[http://www.doe.virginia.gov/testing/solsearch/sol/math/A/math\\_ess\\_a-2b\\_2.pdf](http://www.doe.virginia.gov/testing/solsearch/sol/math/A/math_ess_a-2b_2.pdf)

# The Algebra Tiles Web App



# Our Students Need...





# Questions? Concerns?



# Thank you!

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communications@ged.com*