

# **2018 International Conference Professional Development Day 1, Session 2: Mathematical Reasoning**

## **Information, Resources, and Strategies for the Classroom**

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## Mathematical Practices

1. Make sense of problems and persevere in solving them  
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.



Reasoning and explaining



Modeling and using tools

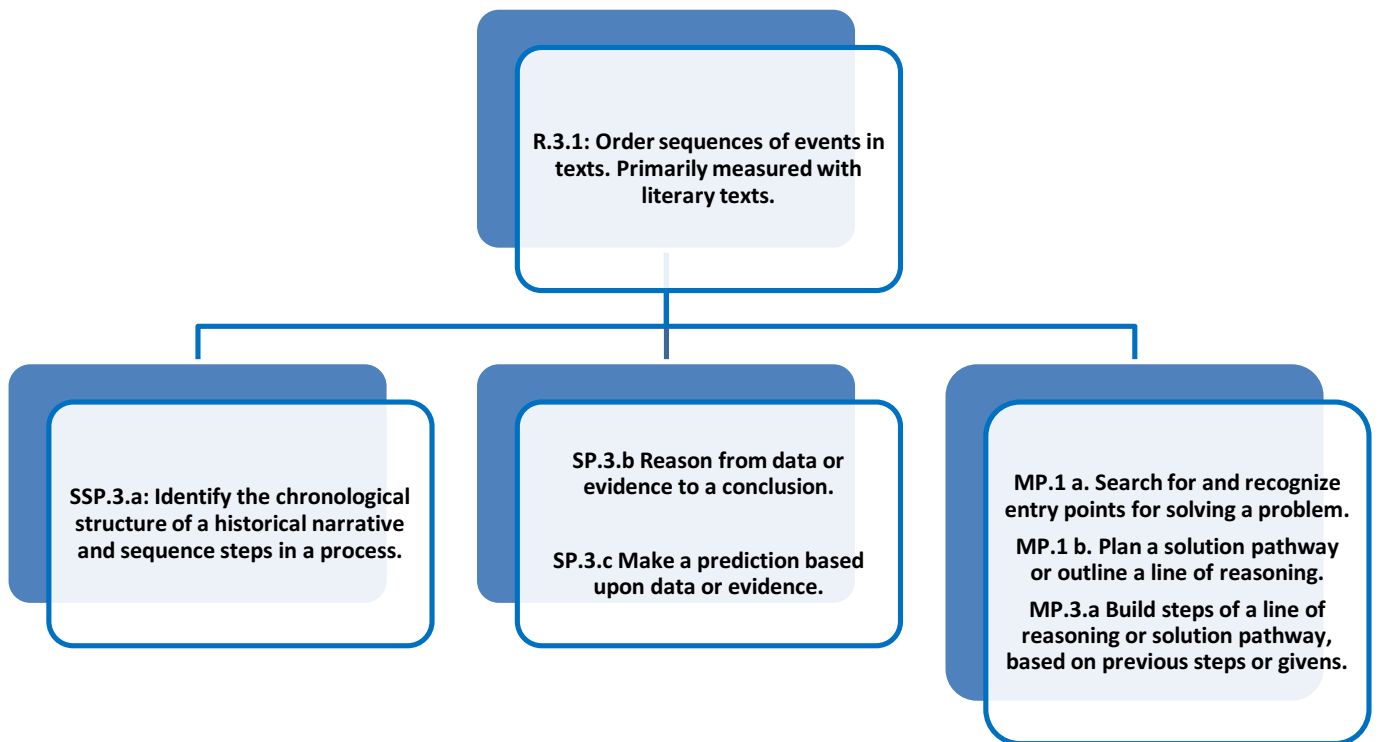


Seeing structure and generalizing



Overarching habits of mind of a productive mathematical thinker.

## High Impact Indicator Relationships





## Reading and Reasoning Process

First Read: Read for Understanding

Second Read: Identify a Problem-Solving Process

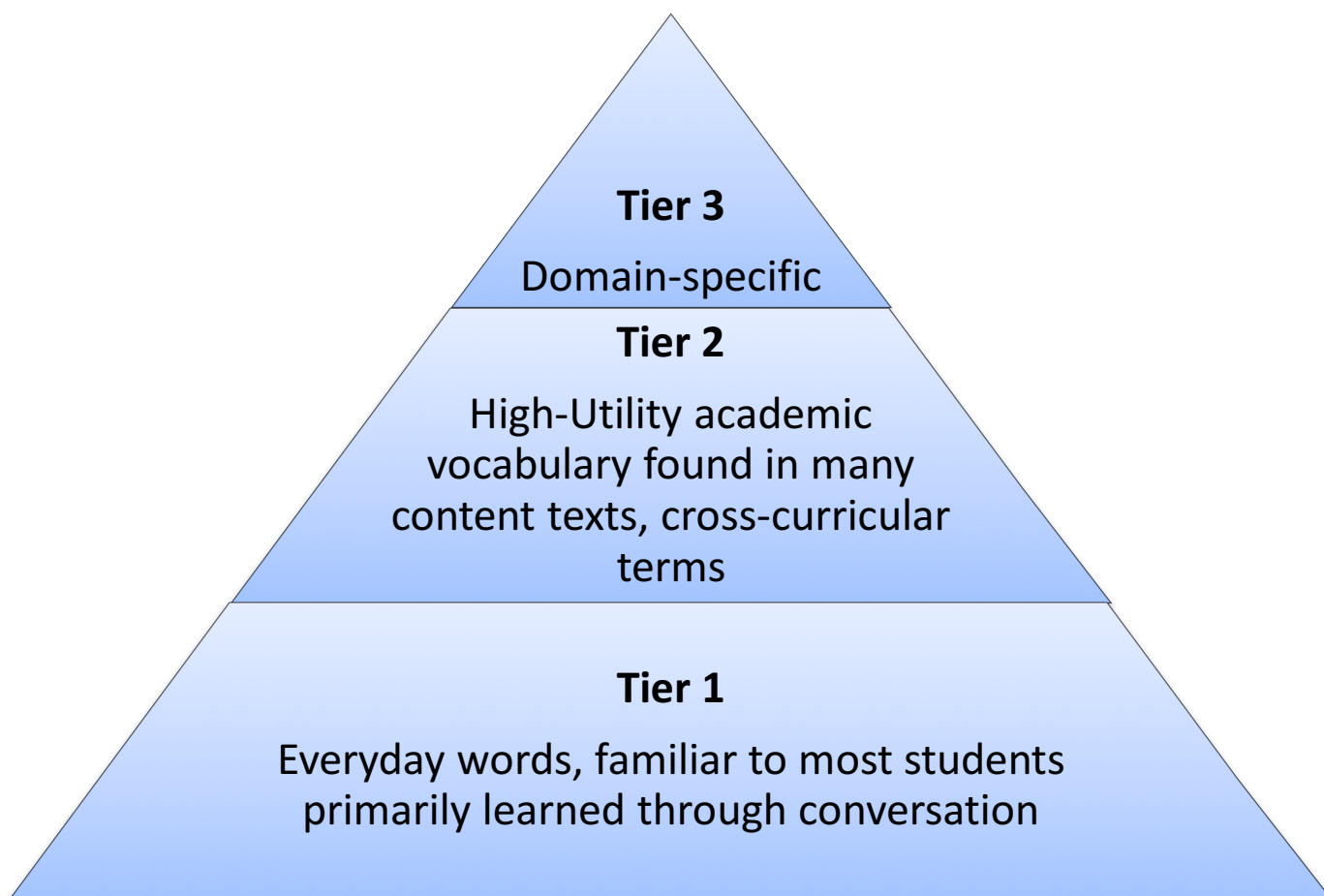
Third Read: Solve the Problem and Check for Reasonableness

## First Read: Read for Understanding



An apartment building contains 12 units consisting of one- and two-bedroom apartments that rent for \$360 and \$450 per month, respectively. When all units are rented, the total monthly rental is \$4,950. What is the number of two-bedroom apartments?

## Tiered Vocabulary



**Frayer Model**

# Framer Model

|                                     |                              |
|-------------------------------------|------------------------------|
| <b>Definition in your own words</b> | <b>Facts/characteristics</b> |
| <b>Examples</b>                     | <b>Nonexamples</b>           |

**Word**

## Second Read – Noticing and Wondering

Mathematical Reasoning - Candidate Name
Question 10 of 16

✓ Answer Explanation
🔍 Calculator
🚩 Flag for Review

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.

**Tree 1**

| Year | Trunk Diameter (inches) |
|------|-------------------------|
| 1    | 18.6                    |
| 3    | 19.2                    |
| 5    | 19.8                    |
| 7    | 20.4                    |
| 9    | 21.0                    |
| 11   | 21.6                    |
| 13   | 22.2                    |

**Tree 2**

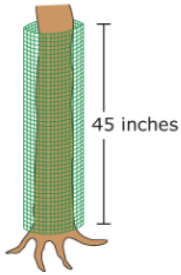
| Year | Trunk Diameter (inches) |
|------|-------------------------|
| 1    | 11.4                    |
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| 5    | 12.6                    |
| 7    | 13.2                    |
| 9    | 13.8                    |
| 11   | 14.4                    |
| 13   | 15.0                    |

This is the final year in which she will collect data. When her data collection is complete, she will predict future red maple tree growth.

📄 Formula Sheet
📄 Calculator Reference

In year 13, the scientist will put tree wrap around tree 1 to protect it from the winter snow. The height of the tree wrap needs to be 45 inches.

**Tree Wrap**



The wrap is priced by the square foot. To the nearest square foot, how many square feet of wrap does she need?

☐ A. 22  
☐ B. 44  
☐ C. 121  
☐ D. 261

← Previous
Next →

What do you notice?

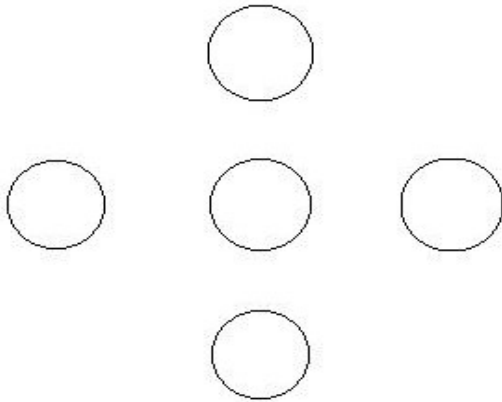
What do you wonder?

## Must Have Heuristics – Thinking Strategies



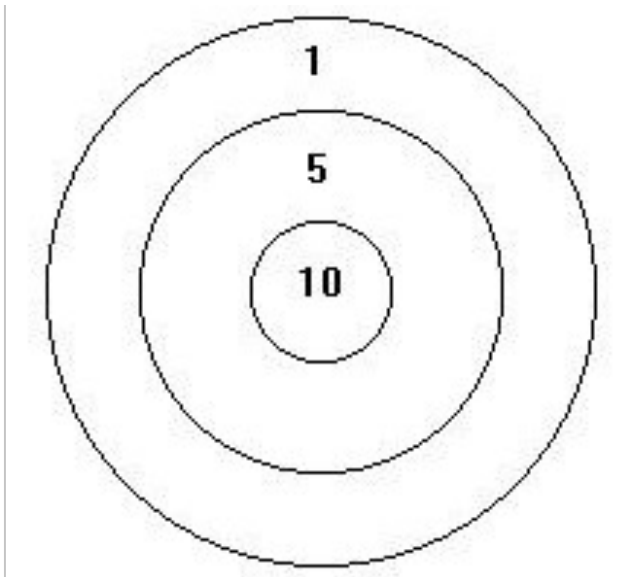
## Guess and Check

Copy the figure below and place the digits 1, 2, 3, 4, and 5 in the circles so that sums across (horizontally) and down (vertically) are the same.



## Make a List

Three darts hit this dart board and each scores a 1, 5, or 10. The total score is the sum of the scores for the three darts. There could be three 1's, two 1's and 5, one 5 and two 10's, and so on. How many different possible total scores could a person get with three darts?



## Draw a Diagram

In a stock car race, the first five finishers in some order were a Ford, a Pontiac, a Chevrolet, a Buick, and a Dodge.

- The Ford finished seven seconds before the Chevrolet.
- The Pontiac finished six seconds after the Buick.
- The Dodge finished eight seconds after the Buick.
- The Chevrolet finished two seconds before the Pontiac.

In what order did the cars finish the race? What strategy did you use?

## Make a Table or Chart

South Point Amusement Park has a special package for large groups: a flat fee of \$20 and \$6 per person.

If a club has \$100 to spend on admission, what is the largest number of people who can attend?



## Find a Pattern

Continue these numerical sequences by finding the next three numbers for each group.

1, 4, 7, 10, 13, \_\_\_\_, \_\_\_\_, \_\_\_\_

19, 20, 22, 25, 29, \_\_\_\_, \_\_\_\_, \_\_\_\_

2, 6, 18, 54, \_\_\_\_, \_\_\_\_, \_\_\_\_

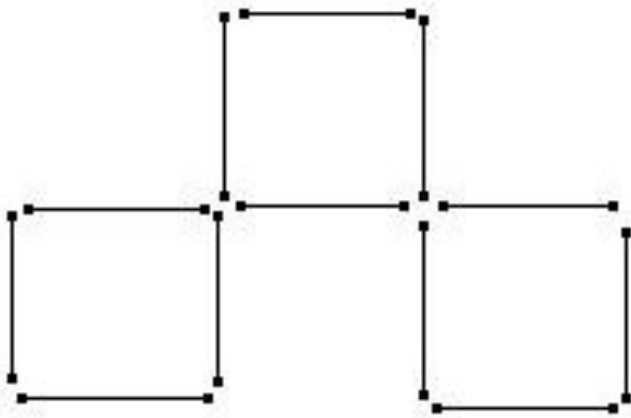
## Make it Simpler

The houses on Main Street are numbered consecutively from 1 to 150.

How many house numbers contain at least one digit 7?

## Act It Out or Use Objects

The figure shows twelve toothpicks arranged to form three squares. How can you form five squares by moving only three toothpicks?



## Work Backwards

Brady was trying to decide when to get up in the morning. He needs 45 minutes to get ready for the workshop he plans to attend. It takes him 25 minutes to drive to the adult center where the workshop will be held. He wanted to get to center 20 minutes early to stop by his classroom and pick up some materials. If the session starts at 7:30, what time should he get up, if he wants to give himself 10 extra minutes in case the traffic is bad?

## Brainstorm and Write an Equation

Two apples weigh the same as a banana and a cherry. A banana weighs the same as nine cherries. How many cherries weigh the same as one apple?

## Use Logical Reasoning

Three apples and two pears cost 78 cents. But two apples and three pears cost 82 cents. What is the total cost of one apple and one pear?

## Problem Solving Graphic Organizer – Goals and Givens

**PROBLEM SOLVING TEMPLATE:** This template can be used as another tool that will develop the process of goals and givens. Students will still have multiple reads of the content to complete this template.

|                                    |   |
|------------------------------------|---|
| <b>Goal: What is the question?</b> | <b>Givens: Important details/information that is provided</b> |
|------------------------------------|---|

**Plan:** What strategies will you use? May have multiple checked.

☐ Draw and Label Diagram/Picture

☐ Look for patterns

☐ Write an equation

☐ Guess and Check

☐ Make a table

☐ Work backwards

☐ Make it simpler

☐ Act out of use  
objects

☐ Other \_\_\_\_\_

Circle the one that was most effective.

**Conjecture:** (reasonable guess)

Predict your answer and any reasoning that results in your predicted answer

**Solution:** (make no assumptions, label everything)

**Solution:** Persevere – if one strategy doesn't work try another one.

**Answer:** Write a complete sentence that answers your goal with appropriate units.

**Verification:** Explain why your answer makes sense. Why is it reasonable? Did you answer the goal? Is there another strategy that proves your answer is correct?

### Scoring Rubric

|                     |                    |              |                   |                       |                 |
|---------------------|--------------------|--------------|-------------------|-----------------------|-----------------|
| Goal/Givens<br>1 pt | Conjecture<br>1 pt | Plan<br>1 pt | Solution<br>2 pts | Verification<br>2 pts | Answer<br>3 pts |
|---------------------|--------------------|--------------|-------------------|-----------------------|-----------------|

## Goals and Givens – It's Your Turn!

Mathematical Reasoning - Candidate Name
Question 10 of 16

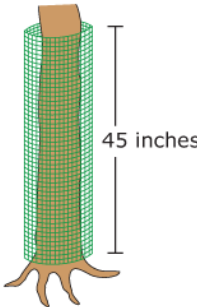
☒ Answer Explanation
☐ Calculator
☐ Flag for Review

Formula Sheet

Calculator Reference

In year 13, the scientist will put tree wrap around tree 1 to protect it from the winter snow. The height of the tree wrap needs to be 45 inches.

**Tree Wrap**



The wrap is priced by the square foot. To the nearest square foot, how many square feet of wrap does she need?

☐ A. 22  
☐ B. 44  
☐ C. 121  
☐ D. 261

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.

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**Tree 2**

| Year | Trunk Diameter (inches) |
|------|-------------------------|
| 1    | 11.4                    |
| 3    | 12.0                    |
| 5    | 12.6                    |
| 7    | 13.2                    |
| 9    | 13.8                    |
| 11   | 14.4                    |
| 13   | 15.0                    |

This is the final year in which she will collect data. When her data collection is complete, she will predict future red maple tree growth.

← Previous
Next →

|                                    |   |
|------------------------------------|---|
| <b>Goal: What is the question?</b> | <b>Givens: Important details/information that is provided</b> |
|------------------------------------|---|

**Plan: What strategies will you use? May have multiple checked.**

☐ Draw and Label Diagram/Picture
☐ Guess and Check
☐ Make it simpler

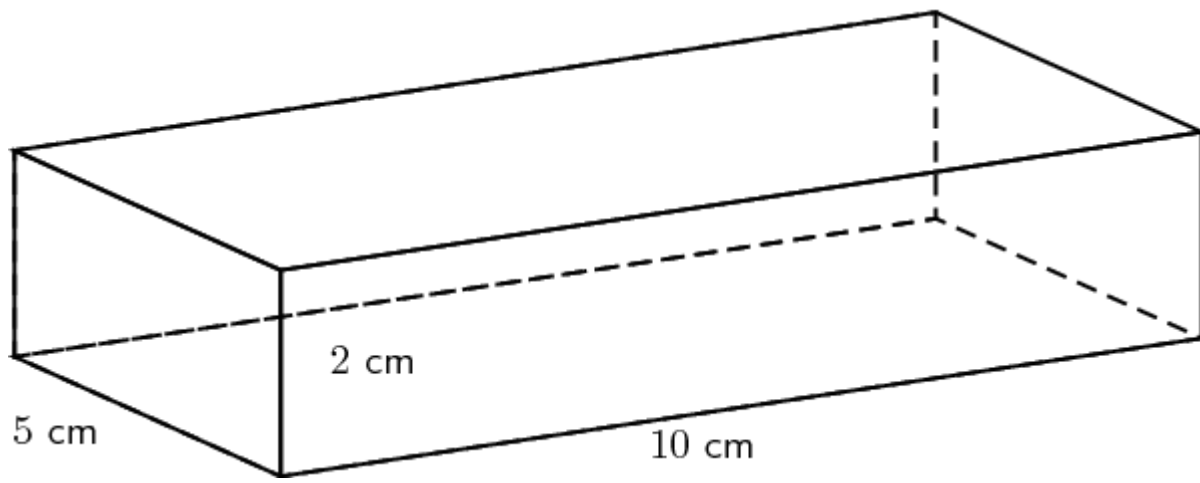
☐ Look for patterns
☐ Make a table
☐ Act out of use objects

☐ Write an equation
☐ Work backwards
☐ Other \_\_\_\_\_

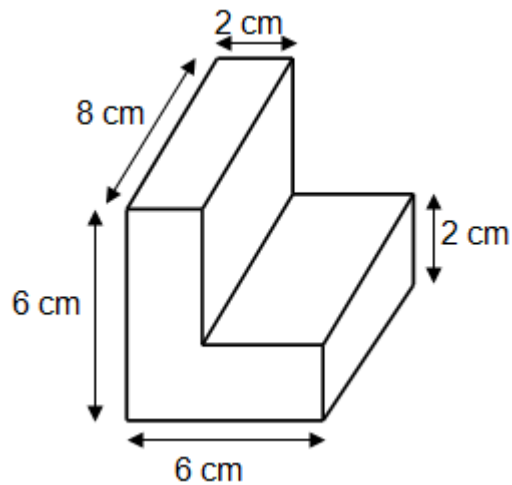
**Quick Draw**

## Using Nets to Find Surface Area

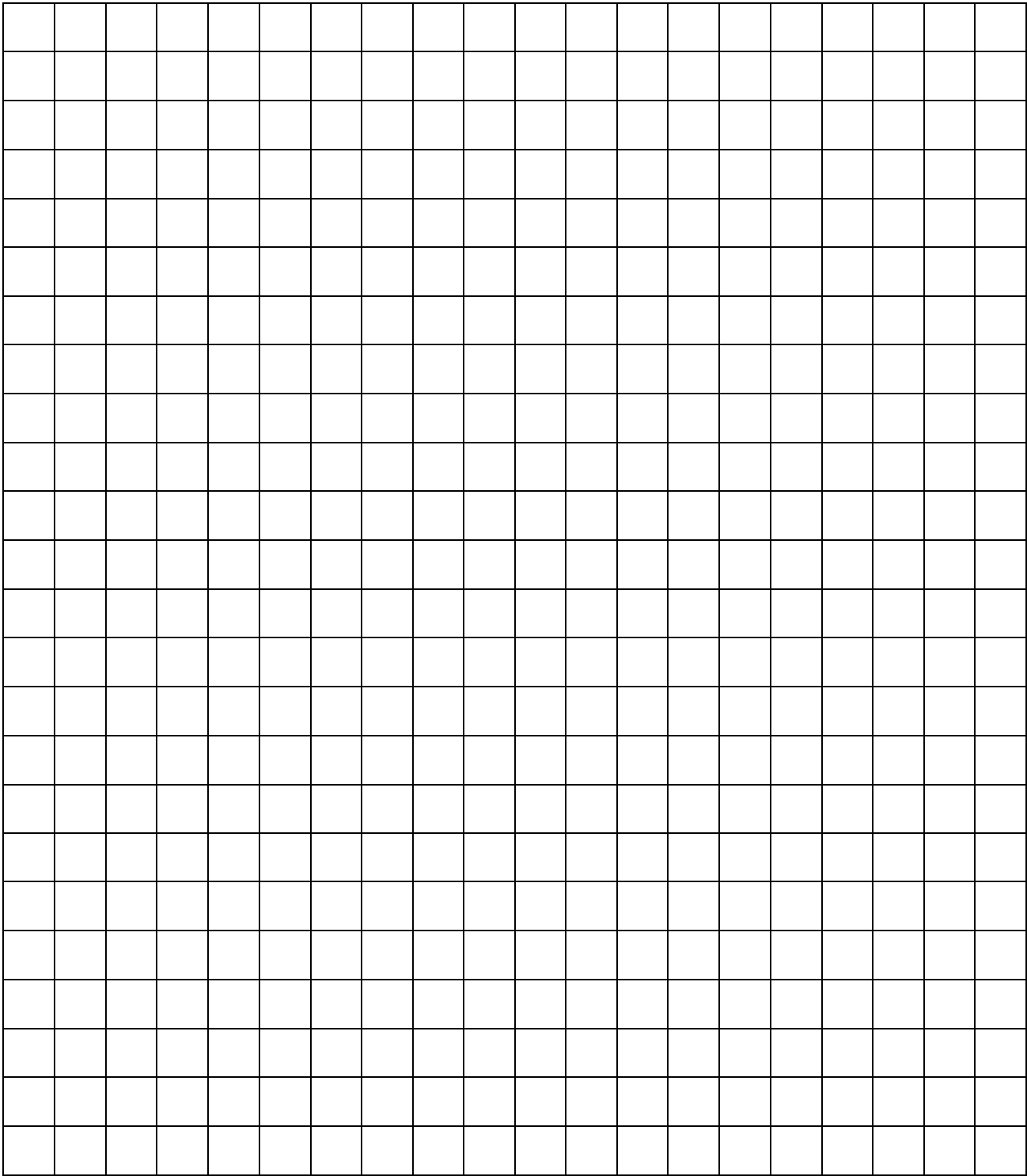
Find the surface area of the rectangular prism by using a net. Use graph paper.



The diagram shows a prism constructed from two rectangular prisms. Draw the net for the solid and mark the lengths. Calculate the surface area of the solid.



**Graph Paper**





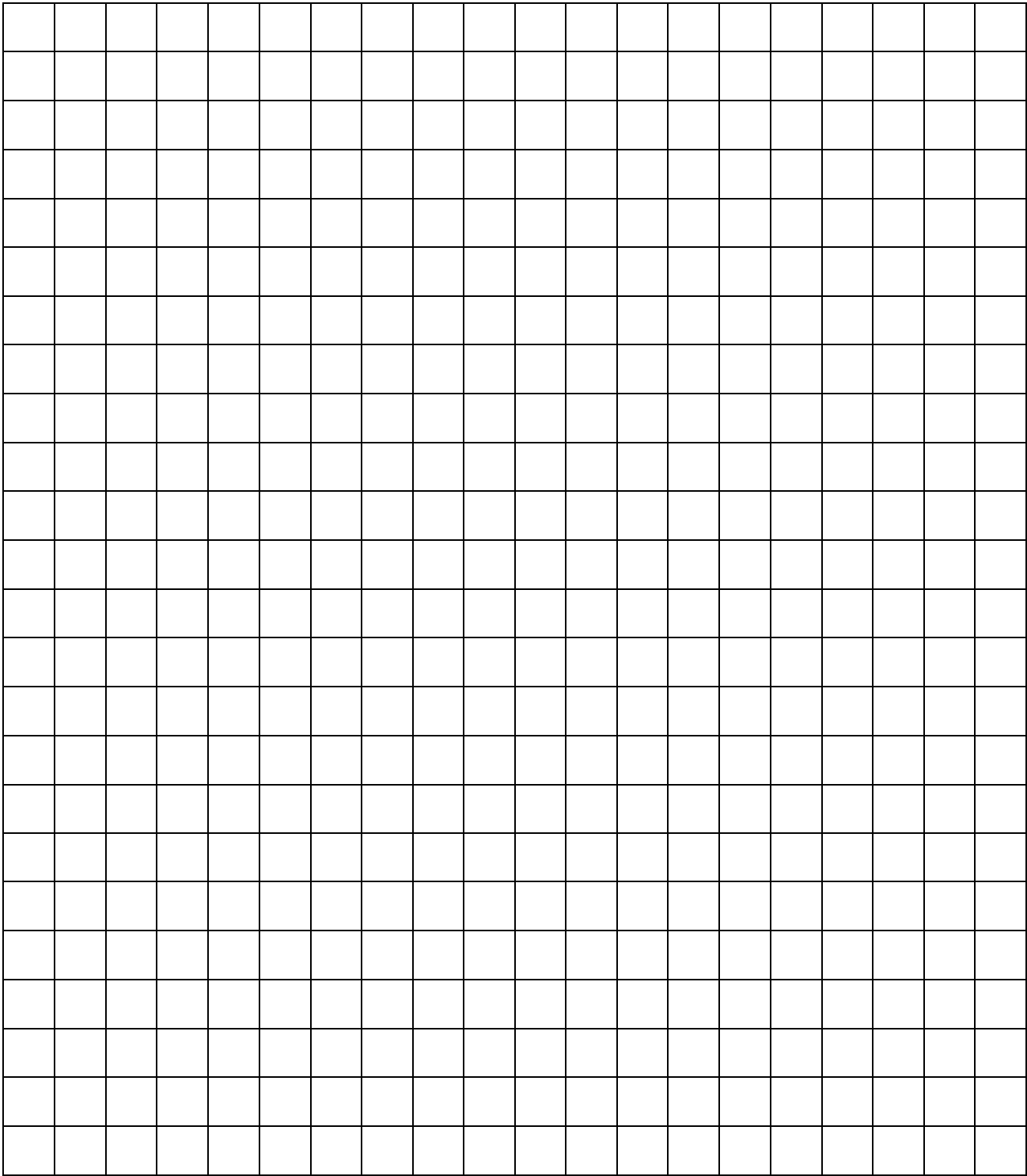
## Build Your Net



I want to paint the outside of a decorative pillar that has a height of 48 inches and a diameter of 16 inches. One small canister of paint will cover about 200 square inches. How many small canisters of paint will I need to paint the cylinder?

- What do I need to know?
- What type of net can I draw?
- What formula can I use?

**Graph Paper**



## What Would Your Students Do?

| Mathematical Reasoning - Candidate Name   |                                     | Question 14 of 16                                       |
|---|-------------------------------------|---|
| <input checked="" type="checkbox"/> Answer Explanation  | <input type="checkbox"/> Calculator | <input type="checkbox"/> Flag for Review                |
| <a href="#">Formula Sheet</a>   |                                     | <a href="#">Calculator Reference</a>                    |
| <p>There are <math>s</math> steps from the pedestal to the head of the Statue of Liberty. The number of steps in the Washington Monument is 27 less than 6 times the number of steps in the Statue of Liberty. Which expression represents the number of steps in the Washington Monument in terms of <math>s</math>?</p> <p><input type="radio"/> A. <math>27 &lt; 6s</math></p> <p><input type="radio"/> B. <math>6(s - 27)</math></p> <p><input type="radio"/> C. <math>6s - 27</math></p> <p><input type="radio"/> D. <math>6s &lt; 27</math></p> |                                     |   |
|   |                                     | <a href="#">&lt; Previous</a> <a href="#">Next &gt;</a> |

## Purposeful Questions

| Question type                            | Description   | Examples   |
|--|---|--|
| Gathering information                    | Students recall facts, definitions, or procedures.  | <ul style="list-style-type: none"> <li>When you write an equation, what does the equal sign tell you?</li> <li>What is the formula for finding the area of a rectangle?</li> </ul>   |
| Probing thinking                         | Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task. | <ul style="list-style-type: none"> <li>As you drew that number line, what decisions did you make so that you could represent <math>\frac{7}{4}</math> on it?</li> <li>Can you show and explain more about how you used a table to find the answer to the Smartphone Plans task?</li> </ul> |
| Making the mathematics visible           | Students discuss mathematical structures and make connections among mathematical ideas and relationships                                  | <ul style="list-style-type: none"> <li>What does your equation have to do with the band concert situation?</li> <li>How does that array relate to multiplication and division?</li> </ul>  |
| Encouraging reflection and justification | Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.         | <ul style="list-style-type: none"> <li>How might you prove that 51 is the solution?</li> <li>How do you know that the sum of two odd numbers will always be even?</li> </ul>   |

## Math Translation Guide

The chart below gives you some of the terms that come up in a lot of word problems. Use them in order to translate or “set-up” word problems into equations.

| English  | Math                           | Example  | Translation   |
|--|--------------------------------|--|---|
| What, a number   | $x, n$ , etc.                  | Three more than a number is 8.   | $n + 3 = 8$   |
| Equivalent, equals, is, was, has, costs                                      | $=$                            | Danny <b>is</b> 16 years old.<br>A CD <b>costs</b> 15 dollars.   | $d = 16$<br>$c = 15$  |
| Is greater than<br>Is less than<br>At least, minimum<br>At most, maximum     | $>$<br>$<$<br>$\geq$<br>$\leq$ | Jenny <b>has more</b> money than Ben.<br>Ashley’s age <b>is less than</b> Nick’s.<br>There are <b>at least</b> 30 questions on the test.<br>Sam can invite <b>a maximum</b> of 15 people to his party. | $j > b$<br>$a < n$<br>$t \geq 30$<br>$s \leq 15$  |
| More, more than, greater, than, added to, total, sum, increased by, together | $+$                            | Kecia has 2 <b>more</b> video games than John.<br>Kecia and John have a <b>total</b> of 11 video games.  | $k = j + 2$<br>$k + j = 11$   |
| Less than, smaller than, decreased by, difference, fewer                     | $-$                            | Jason has 3 <b>fewer</b> CDs than Carson.<br>The <b>difference between</b> Jenny’s and Ben’s savings is \$75.  | $j = c - 3$<br>$j - b = 75$   |
| Of, times, product of, twice, double, triple, half of, quarter of            | $\times$                       | Emma has <b>twice</b> as many books as Justin.<br><br>Justin has <b>half</b> as many books as Emma.  | $e = 2 \times j$<br>or<br>$e = 2j$<br><br>$j = c \times \frac{1}{2}$<br>or<br>$j = e/2$ |
| Divided by, per, for, out of, ratio of ___ to ___                            | $\div$                         | Sophia has \$1 <b>for</b> every \$2 Daniel has.<br><br>The <b>ratio of</b> Daniel’s savings <b>to</b> Sophia’s savings is 2 to 1.  | $s = d \div 2$<br>or<br>$s = d/2$<br><br>$d/s = 2/1$                                    |

### Sample

Jennifer has 10 fewer DVDs than Brad.

Step 1:  $j$  (has) =  $b$  (fewer) – 10

Remember, the word “has” is an equal sign and the word “fewer” is a minus sign, so:

Step 2:  $j = b - 10$

### Example 1

Clay got 10 fewer votes than Kimberly. Reuben got three times as many votes as Clay. The three contestants received a total of 90 votes. Write an equation in one variable that can be used to solve for the number of votes Kimberly received.

**Step 1:** Pick which unknown will be represented by the variable. Since you're solving for Kimberly, let  $k$  be the number of votes Kimberly received.

**Step 2:** Represent the other two unknowns in terms of  $k$ . Clay got 10 fewer votes so it's  $k - 10$  and Reuben got three times that so it's  $3(k - 10)$ .

**Step 3:** Set up the equation using all of the expressions to equal 90.

$$k + (k - 10) + 3(k - 10) = 90$$

### Example 2

A school is having a special event to honor successful alumni. The event will cost \$500, plus an additional \$85 for each alum who is honored. Write an equation that best represents the number of alumni that can be honored.

**Step 1:** The amount the school can spend is equal to or less than \$1,000, so it's  $\leq 1,000$

**Step 2:** The event has a fixed cost of \$500 and a variable of \$85 per alum so it's  $500 + 85a$ .

**Step 3:** The equation then becomes  $500 + 85a \leq 1,000$ .

### Example 3

A computer repair company charges \$50 for a service call plus \$25 for each hour of work. Write an equation that represents the relationship between the bill,  $b$ , for a service call, and the number of hours spent on the call,  $h$ .

**Step 1:** Some questions include a situation where there is more than one cost. One of them is fixed and one is variable. First identify the sum of the fixed and variable costs so  $b$  equals the total.

**Step 2:** Next, identify the fixed cost of 50 and the variable cost of  $25h$  (25 x the number of hours).

**Step 3:** The equation then becomes  $50 + 25h = b$ .

# Research and Materials

## Frayer Model

# Frayer Model

|                                     |                              |
|-------------------------------------|------------------------------|
| <b>Definition in your own words</b> | <b>Facts/characteristics</b> |
| <b>Examples</b>                     | <b>Nonexamples</b>           |

**Word**



# Helpful Problem-Solving Heuristics<sup>1</sup>

A *heuristic* is a thinking strategy, something that can be used to tease out further information about a problem and thus help you figure out what to do when you don't know what to do. Here are 25 heuristics that can be useful in solving problems. They help you monitor your thought processes, to step back and watch yourself at work, and thus keep your cool in a challenging situation.

## Group A

1. Ask somebody else how to do the problem. This strategy is probably the most used world-wide, though it is not one we encourage our students to use, at least not initially.
2. Guess and try (guess, check, and revise). Your first guess might be right! But incorrect guesses can often suggest a direction toward a solution. (A spreadsheet is a powerful aid in guessing and trying. Set up the relationships and plug in a number to see if you get what you want. If you don't, it is easy to try another number. And another.)

## Group B

3. Restate the problem using words that make sense to you. One way to do this is to explain the problem to someone else. Often this is all it takes for the light to dawn.
4. Organize information into a table or chart. Having it laid out clearly in front of you frees up your mind for thinking. And perhaps you can use the organized data to generate more information.
5. Draw a picture of the problem. Translate problem information into pictures, diagrams, sketches, glyphs, arrows, or some other kind of representation.
6. Make a model of the problem. The model might be a physical or mental model, perhaps using a computer. You might vary the problem information to see whether and how the model may be affected.
7. Look for patterns, any kind of patterns: number patterns, verbal patterns, spatial/visual patterns, patterns in time, patterns in sound. (Some people define mathematics as the science of patterns.)
8. Act out the problem, if it is stated in a narrative form. Acting it out can have the same effect as drawing a picture. What's more, acting out the problem might disclose incorrect assumptions you are making.
9. Invent notation. Name things in the problem (known or unknown) using words or symbols, including relationships between problem components.
10. Write equations. An equation is simply the same thing named two different ways.

## Group C

11. Check all possibilities in a systematic way. A table or chart may help you to be systematic.
12. Work backwards from the end condition to the beginning condition. Working backwards is particularly helpful when letting a variable (letter) represent an unknown.
13. Identify subgoals in the problem. Break up the problem into a sequence of smaller problems ("If I knew this, then I could get that").
14. Simplify the problem. Use easier or smaller numbers, or look at extreme cases (e.g., use the minimum or maximum value of one of the varying quantities).

## Group D

15. Restate the problem again. After working on the problem for a time, back off a bit and put it into your own words in still a different way, since now you know more about it.
16. Change your point of view. Use your imagination to change the way you are looking at the problem. Turn it upside down, or pull it inside out.
17. Check for hidden assumptions you may be making (you might be making the problem harder than it really is). These assumptions are often found by changing the given numbers or conditions and looking to see what happens.
18. Identify needed and given information clearly. You may not need to find everything you think you need to find, for instance.
19. Make up your own technique. It is your mind, after all; use mental actions that make sense to you. The key is to do something that engages you with the problem.
20. Try combinations of the above heuristics.

These heuristics can be readily pointed out to students as they engage problems in the classroom. However, real-world problems are often confronted many times over or on increasingly complex levels. For those kinds of problems, George Polya, the father of modern problem-solving heuristics, identified a fifth class (E) of looking-back heuristics. We include these here for completeness, but also with the teaching caveat that solutions often improve and insights grow deeper after the initial pressure to produce a solution has been resolved. Subsequent considerations of a problem situation are invariably deeper than the first attempt.

## Group E

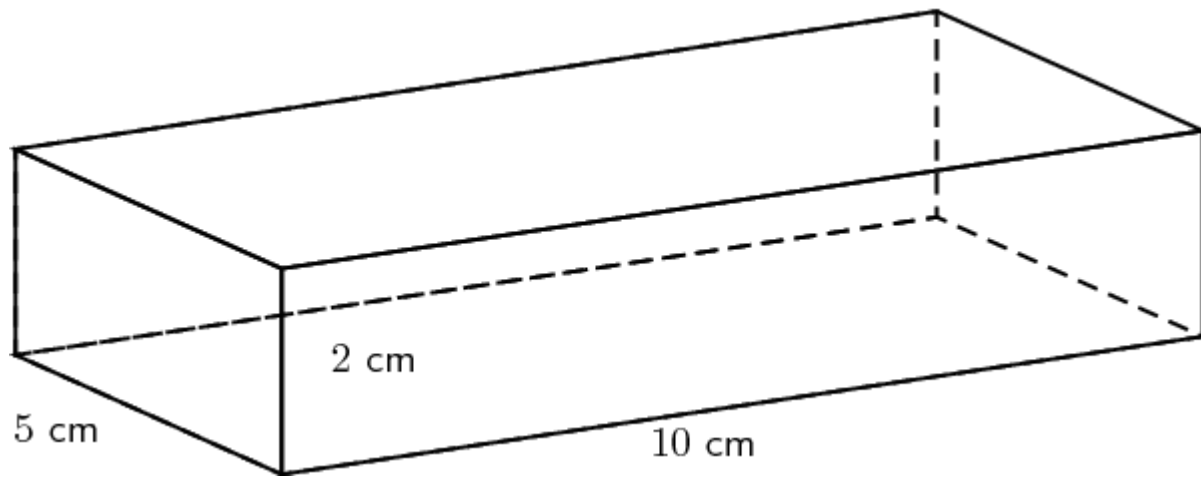
1. Check your solution. Substitute your answer or results back into the problem. Are all of the conditions satisfied?
2. Find another solution. There may be more than one answer. Make sure you have them all.
3. Solve the problem a different way. Your first solution will seldom be the best solution. Now that the pressure is off, you may readily find other ways to solve the problem.
4. Solve a related problem. Steve Brown and Marion Walter in their book, *The Art of Problem Posing*, suggest the "What if not?" technique. What if the train goes at a different speed? What if there are 8 children, instead of 9? What if . . . ? Fascinating discoveries can be made in this way, leading to a solution.
5. Generalize the solution. Can you glean from your solution how it can be made to fit a whole class of related situations? Can you prove your result?

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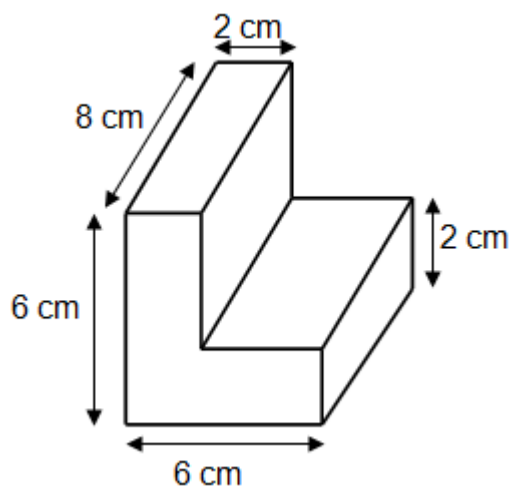
<sup>1</sup> Adapted from Meiring, S. P. (1980). *Problem solving — A basic mathematics goal*. Columbus: Ohio Department of Education.

## Using Nets to Find Surface Area

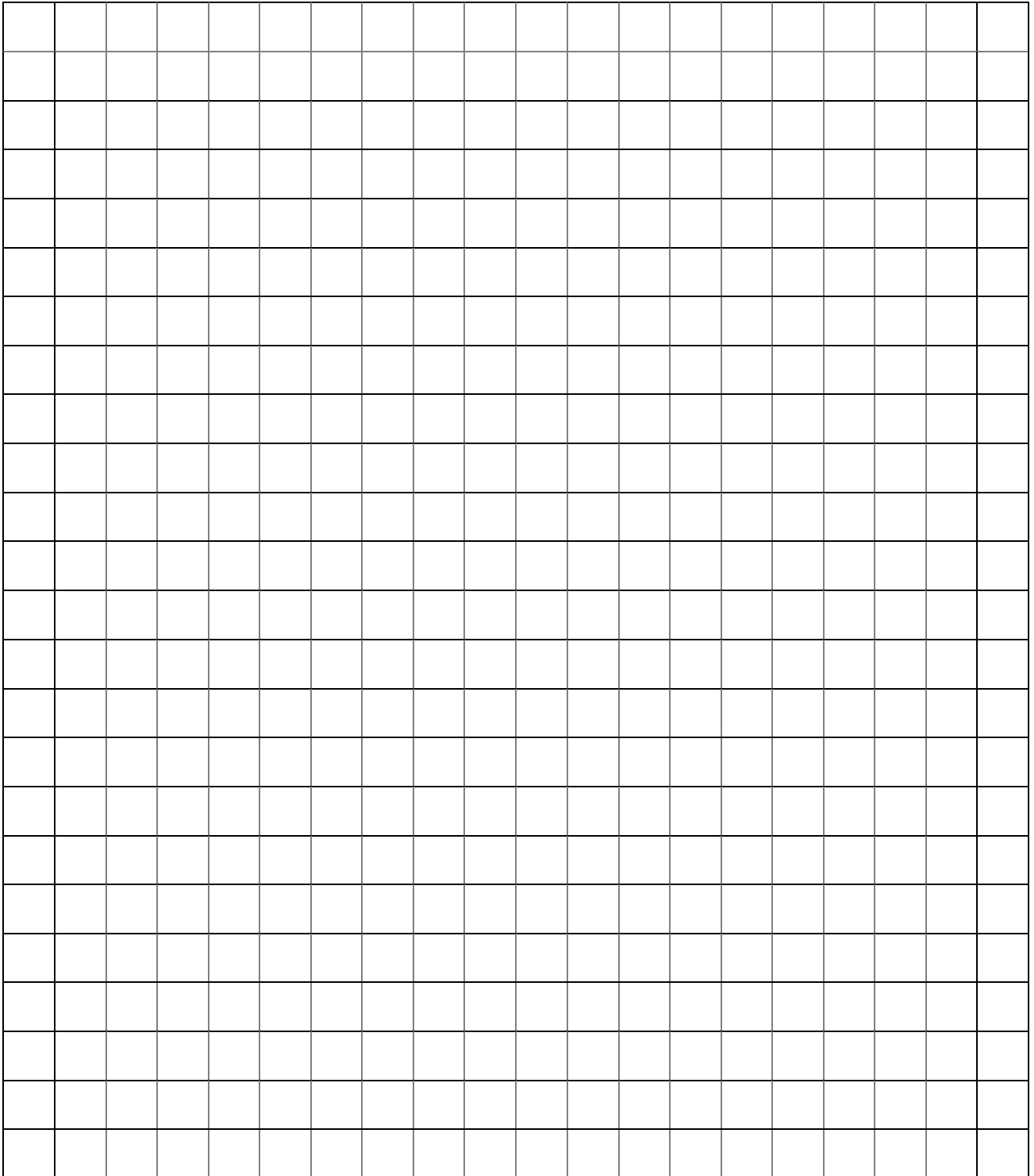
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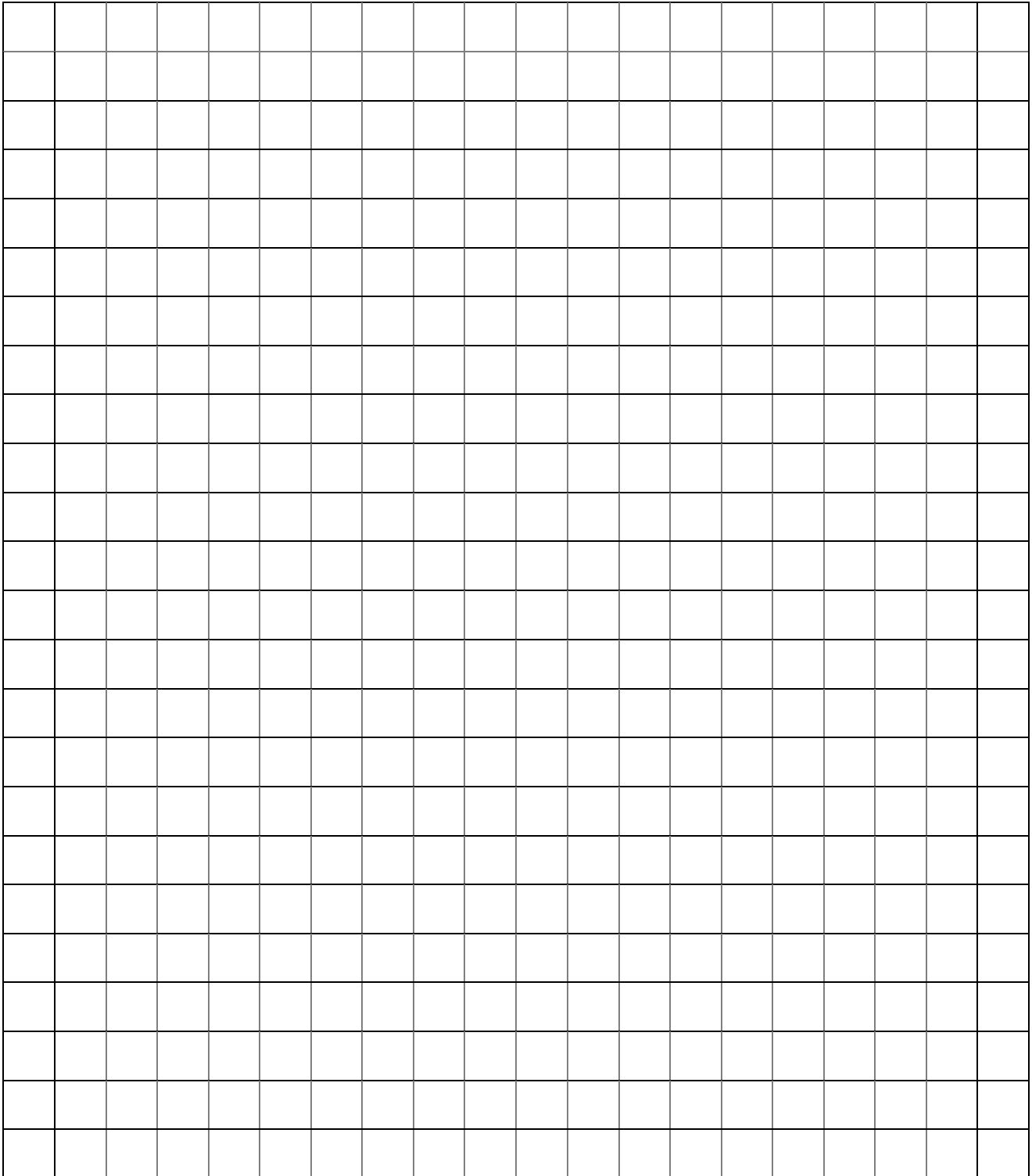
The diagram shows a prism constructed from two rectangular prisms. Draw the net for the solid and mark the lengths. Calculate the surface area of the solid.



## Graph Paper



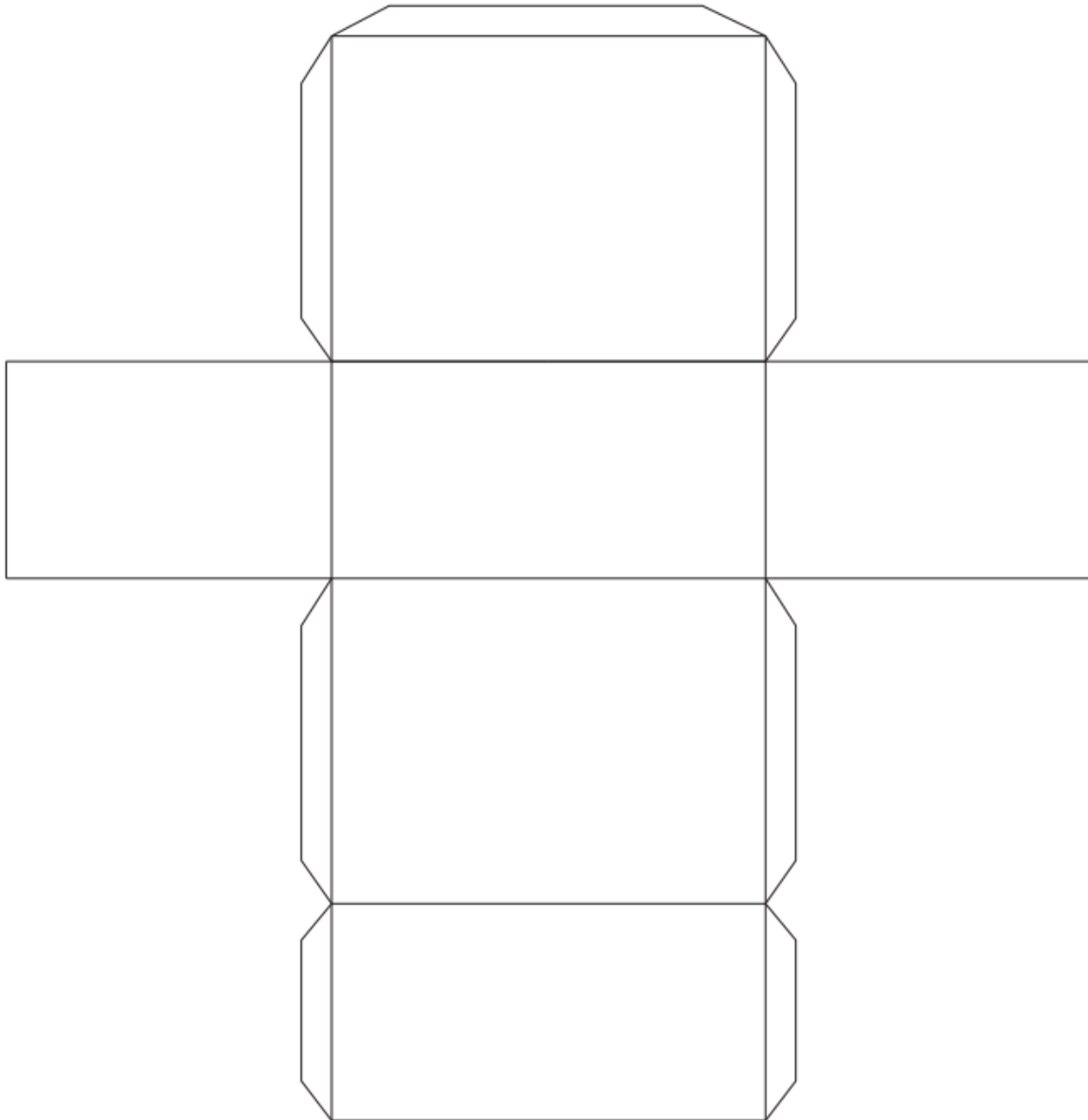
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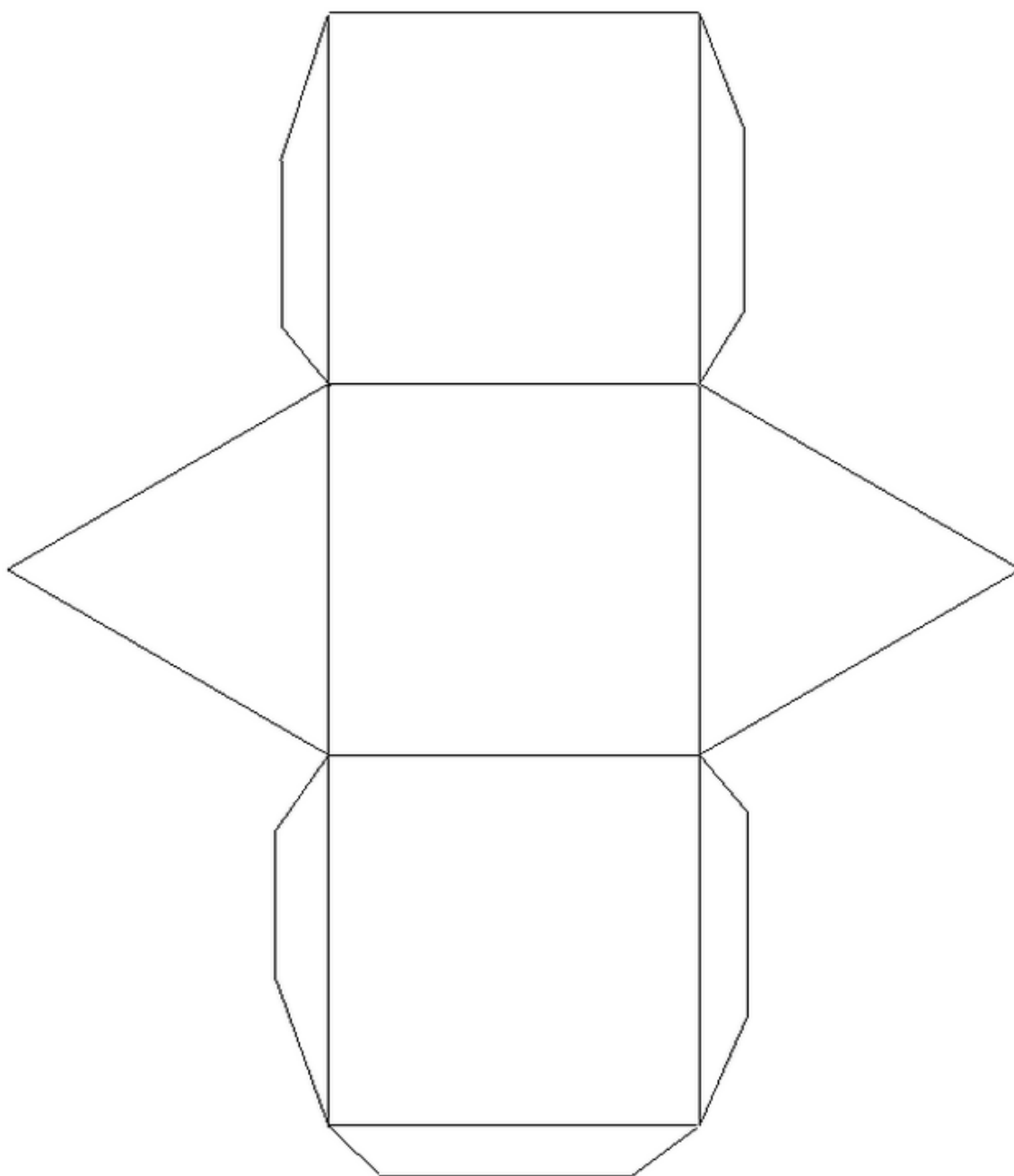
## Sample 3D Nets

Cut out the figures and put them together. Show students how they can unfold the three-dimensional shape into a two-dimensional shape. Tabs are provided for assistance in “putting” the shape together. You may wish to access other shapes through a search on the Internet.

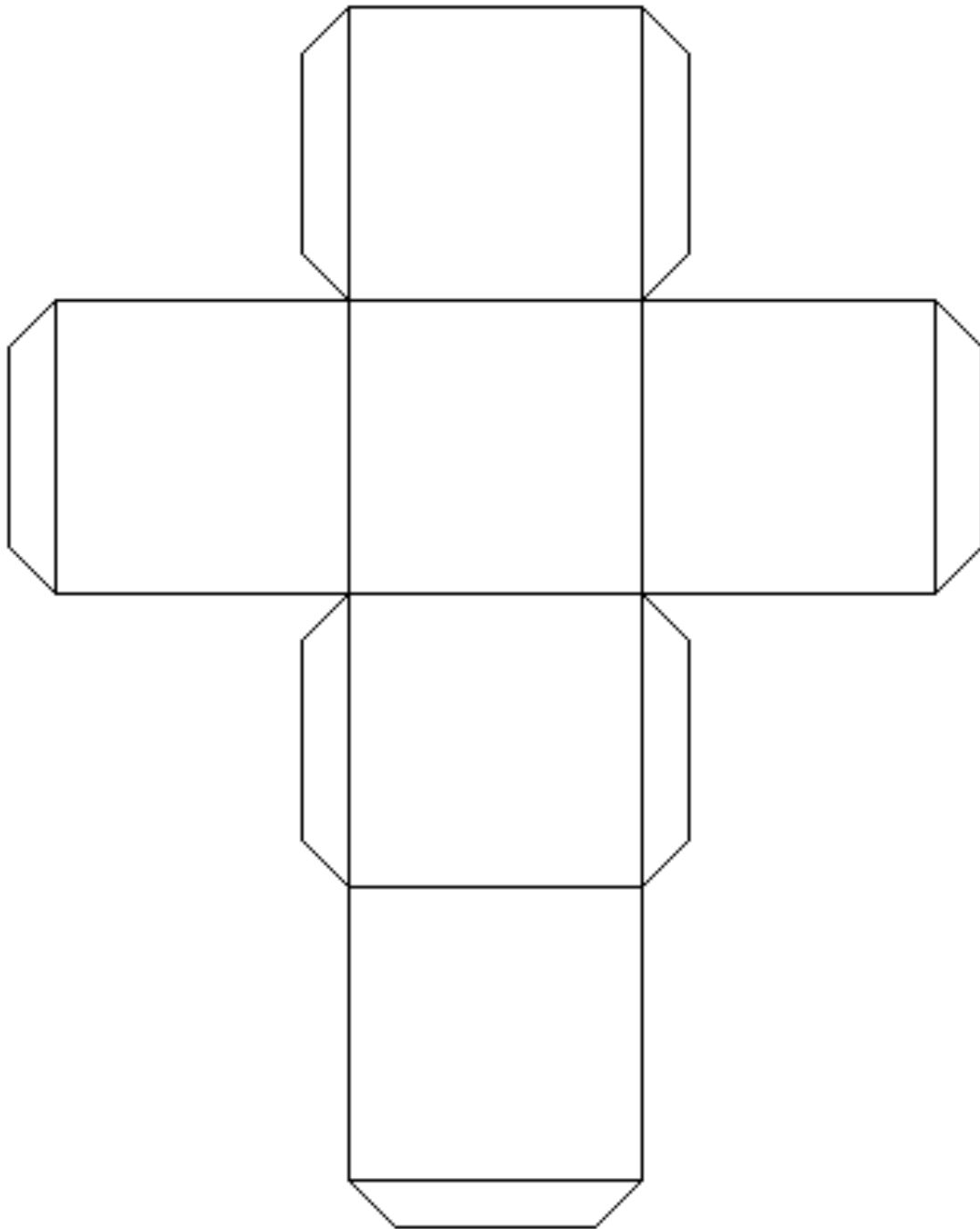
### Rectangular Prism



## Triangular Prism

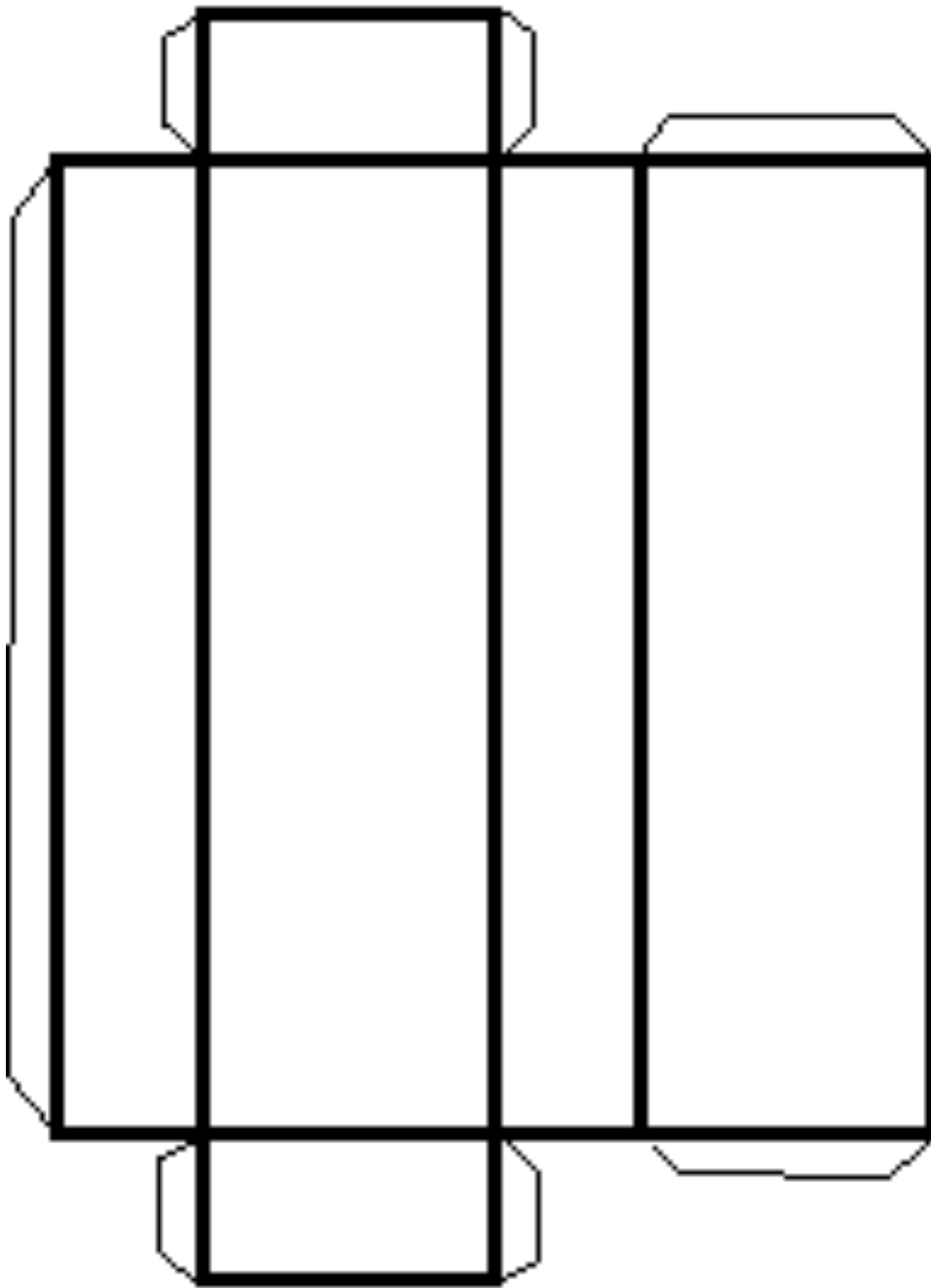


## Cube

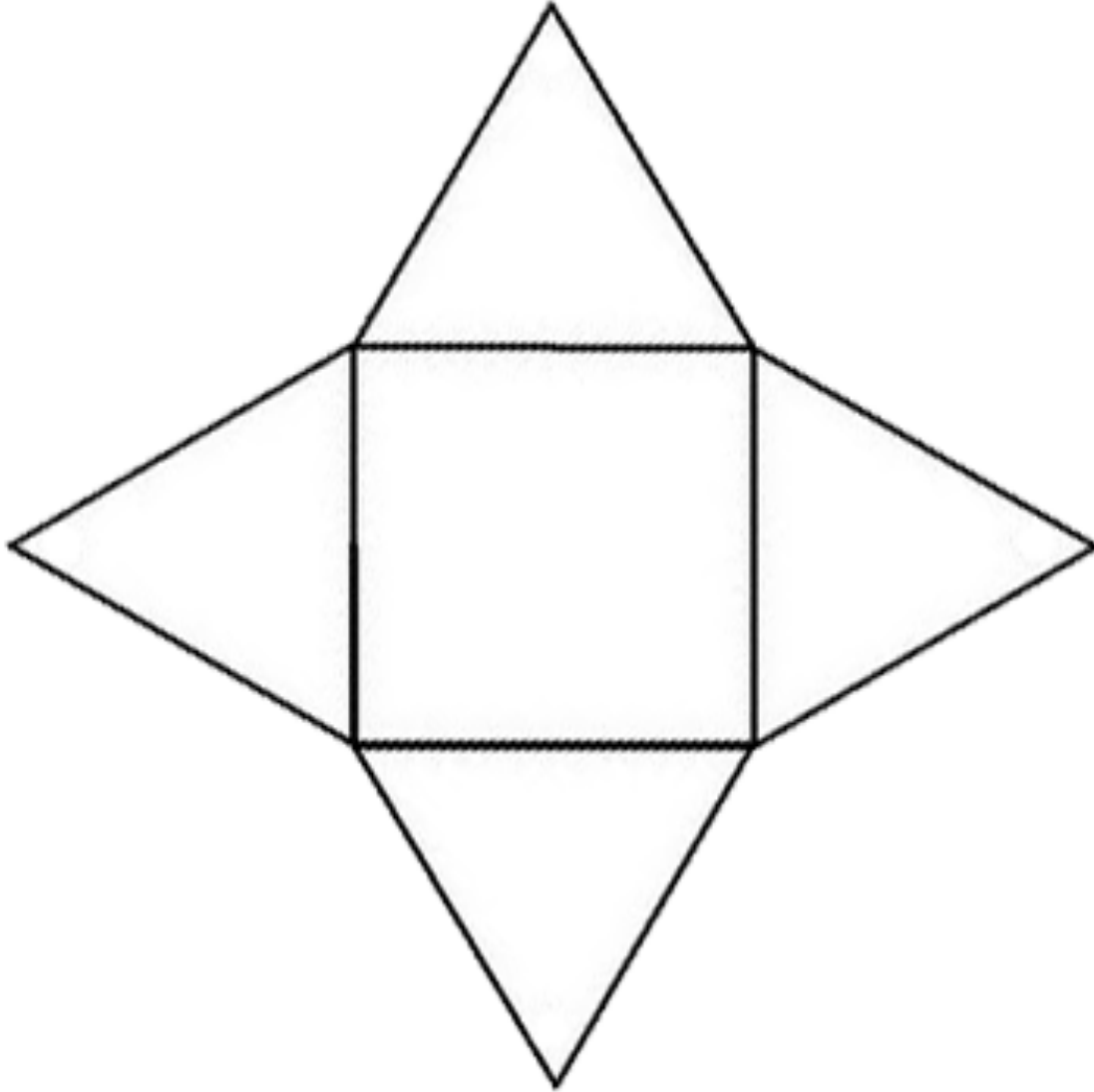




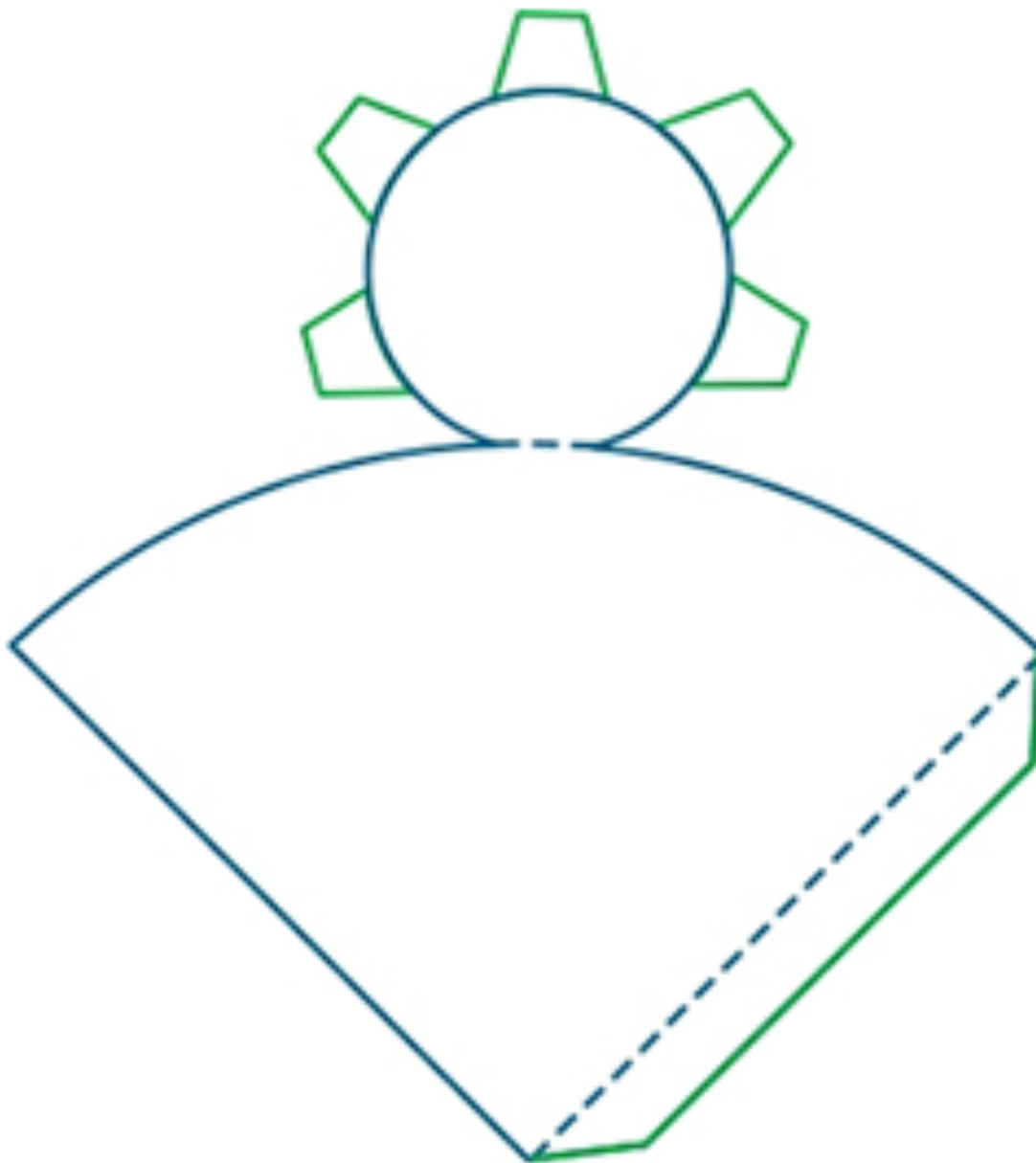
## Cuboid



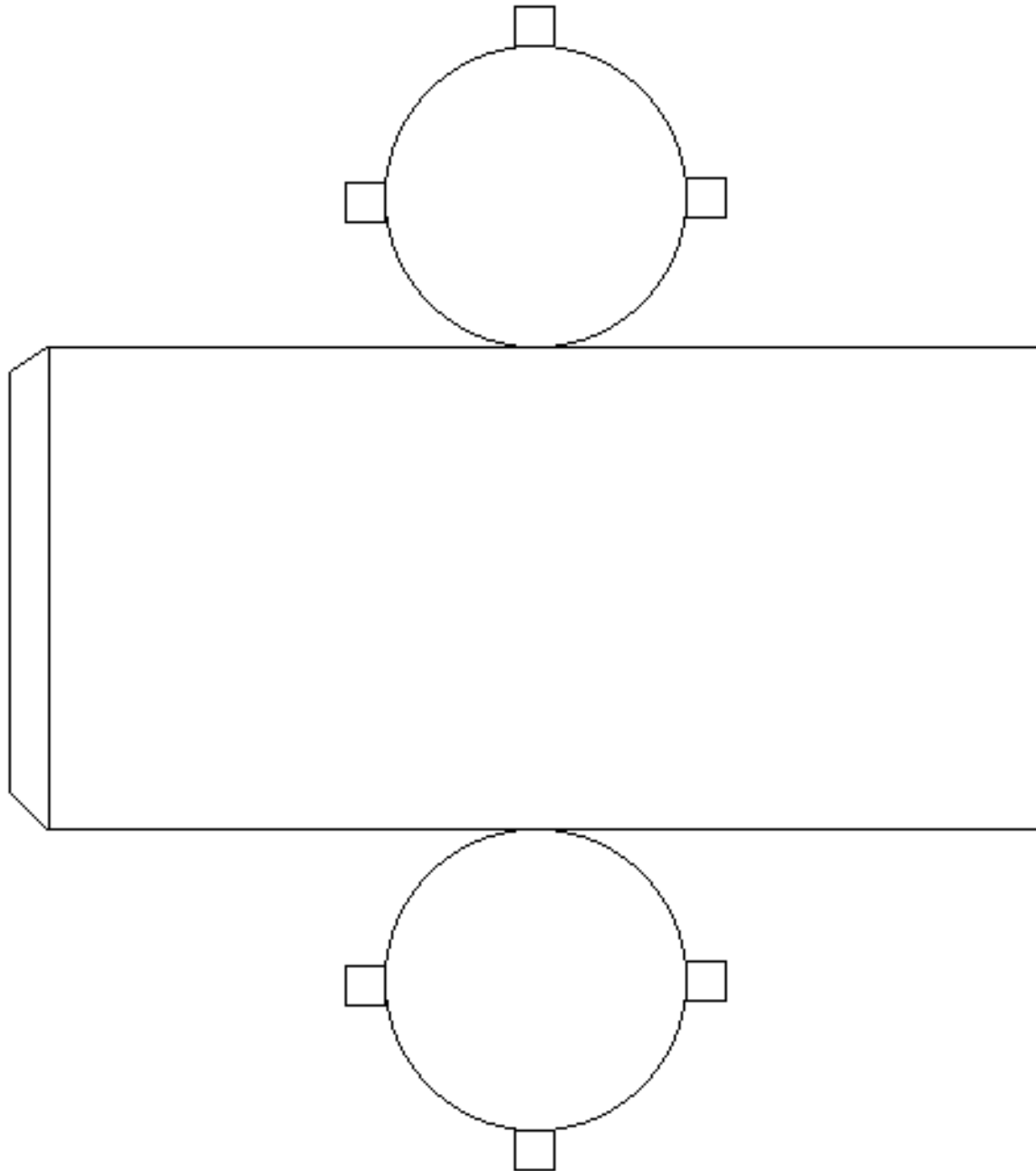
**Pyramid (Square Based)**



## Cone



## Cylinder



## Problem Solving Graphic Organizer – Goals and Givens

**PROBLEM SOLVING TEMPLATE:** This template can be used as another tool that will develop the process of goals and givens. Students will still have multiple reads of the content to complete this template.

|                             |  |
|-----------------------------|--|
| Goal: What is the question? | Givens: Important details/information that is provided |
|-----------------------------|--|

**Plan:** What strategies will you use? May have multiple checked.

- |   |  |  |
|---|--|--|
| <input type="checkbox"/> Draw and Label Diagram/Picture | <input type="checkbox"/> Look for patterns         | <input type="checkbox"/> Write an equation |
| <input type="checkbox"/> Guess and Check                | <input type="checkbox"/> Make a table              | <input type="checkbox"/> Work backwards    |
| <input type="checkbox"/> Make it simpler                | <input type="checkbox"/> Act out of use<br>objects | <input type="checkbox"/> Other _____       |

Circle the one that was most effective.

**Conjecture:** (reasonable guess)

Predict your answer and any reasoning that results in your predicted answer

**Solution:** (make no assumptions, label everything)

**Solution:** Persevere – if one strategy doesn't work try another one.

**Answer:** Write a complete sentence that answers your goal with appropriate units.

**Verification:** Explain why your answer makes sense. Why is it reasonable? Did you answer the goal? Is there another strategy that proves your answer is correct?

### Scoring Rubric

|                     |                    |              |                   |                       |                 |
|---------------------|--------------------|--------------|-------------------|-----------------------|-----------------|
| Goal/Givens<br>1 pt | Conjecture<br>1 pt | Plan<br>1 pt | Solution<br>2 pts | Verification<br>2 pts | Answer<br>3 pts |
|---------------------|--------------------|--------------|-------------------|-----------------------|-----------------|

## Math Translation Guide

The chart below gives you some of the terms that come up in a lot of word problems. Use them in order to translate or “set-up” word problems into equations.

| English  | Math                           | Example  | Translation   |
|--|--------------------------------|--|---|
| What, a number   | $x, n$ , etc.                  | Three more than a number is 8.   | $N + 3 = 8$   |
| Equivalent,<br>equals, is, was,<br>has, costs  | $=$                            | Danny is 16 years old.<br>A CD costs 15 dollars.   | $d = 16$<br>$c = 15$  |
| Is greater than<br>Is less than<br>At least, minimum<br>At most, maximum                 | $>$<br>$<$<br>$\geq$<br>$\leq$ | Jenny has more money than Ben.<br>Ashley's age is less than Nick's.<br>There are at least 30 questions on the test.<br>Sam can invite a maximum of 15 people to his party. | $j > b$<br>$a < n$<br>$t \geq 30$<br>$s \leq 15$                                    |
| More, more than,<br>greater, than,<br>added to, total,<br>sum, increased<br>by, together | $+$                            | Kecia has 2 more video games than John.<br><br>Kecia and John have a total of 11 video games.  | $k = j + 2$<br><br>$k + j = 11$   |
| Less than, smaller<br>than, decreased<br>by, difference,<br>fewer                        | $-$                            | Jason has 3 fewer CDs than Carson.<br>The difference between Jenny's and Ben's savings is \$75.  | $j = c - 3$<br>$j - b = 75$   |
| Of, times, product<br>of, twice, double,<br>triple, half of,<br>quarter of               | $\times$                       | Emma has twice as many books as Justin.<br><br>Justin has half as many books as Emma.  | $e = 2 \times j$<br>or<br>$e = 2j$<br>$j = c \times \frac{1}{2}$<br>or<br>$j = e/2$ |
| Divided by, per,<br>for, out of, ratio of<br>___ to ___                                  | $\div$                         | Sophia has \$1 for every \$2 Daniel has.<br><br>The ratio of Daniel's savings to Sophia's savings is 2 to 1.   | $s = d \div 2$<br>or<br>$s = d/2$<br>$d/s = 2/1$                                    |

**Example 1**

Jennifer has 10 fewer DVDs than Brad.

Step 1:  $j$  (has) =  $b$  (fewer) – 10

Remember, the word “has” is an equal sign and the word “fewer” is a minus sign, so:

Step 2:  $j = b - 10$

**Example 2**

Clay got 1- fewer votes than Kimberly. Reuben got three times as many votes as Clay.

The three contestants received a total of 90 votes. Write an equation in one variable that can be used to solve for the number of votes Kimberly received.

Step 1: Pick which unknown will be represented by the variable. Since you’re solving for Kimberly, let  $k$  be the number of votes Kimberly received.

Step 2: Represent the other two unknowns in terms of  $k$ . Clay got 10 fewer votes so it’s  $k - 10$  and Reuben got three times that so it’s  $3(k - 10)$ .

Step 3: Set up the equation using all of the expressions to equal 90.

$$k + (k - 10) + 3(k - 10) = 90$$

**Example 3:**

A school is having a special even to honor successful alumni. The event will cost \$500, plus an additional \$85 for each alum who is honored. Write an equation that best represents the number of alumni that can be honored.

Step 1: The amount the school can spend is equal to or less than \$1,000, so it’s  $\leq 1,000$

Step 2: The event has a fixed cost of \$500 and a variable of \$85 per alum so it’s  $500 + 85a$ .

Step 3: The equation then becomes  $500 + 85a \leq 1,000$ .

**Example 4:**

A computer repair company charges \$50 for a service call plus \$25 for each hour of work. Write an equation that represents the relationship between the bill,  $b$ , for a service call, and the number of hours spent on the call,  $h$ .

- Step 1: Some questions include a situation where there is more than one cost. One of them is fixed and one is variable. First identify the sum of the fixed and variable costs so  $b$  equals the total.
- Step 2: Next, identify the fixed cost of 50 and the variable cost of  $25h$  (25 x the number of hours).
- Step 3: The equation then becomes  $50 + 25h = b$ .



# Problem Solving Strategies from George Polya

George Polya (1887 – 1985) was one of the most famous mathematics educators of the 20<sup>th</sup> century (so famous that you probably never even heard of him). Dr. Polya strongly believed that the skill of problem solving could and should be taught – it is not something that you are born with. He identifies four principles that form the basis for any serious attempt at problem solving:

- 1. Understand the problem**
- 2. Devise a plan**
- 3. Carry out the plan**
- 4. Look back (reflect)**

## 1. Understand the problem

- What are you asked to find out or show?
- Can you draw a picture or diagram to help you understand the problem?
- Can you restate the problem in your own words?
- Can you work out some numerical examples that would help make the problem clearer?

## 2. Devise a plan

A partial list of Problem Solving Strategies includes:

|                                  |                                  |
|----------------------------------|----------------------------------|
| <i>Guess and check</i>           | <i>Solve a simpler problem</i>   |
| <i>Make an organized list</i>    | <i>Experiment</i>                |
| <i>Draw a picture or diagram</i> | <i>Act it out</i>                |
| <i>Look for a pattern</i>        | <i>Work backwards</i>            |
| <i>Make a table</i>              | <i>Use deduction</i>             |
| <i>Use a variable</i>            | <i>Change your point of view</i> |

## 3. Carry out the plan

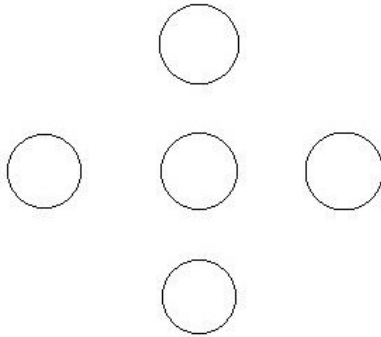
- Carrying out the plan is usually easier than devising the plan
- Be patient – most problems are not solved quickly nor on the first attempt
- If a plan does not work immediately, be persistent
- Do not let yourself get discouraged
- If one strategy isn't working, try a different one

## 4. Look back (reflect)

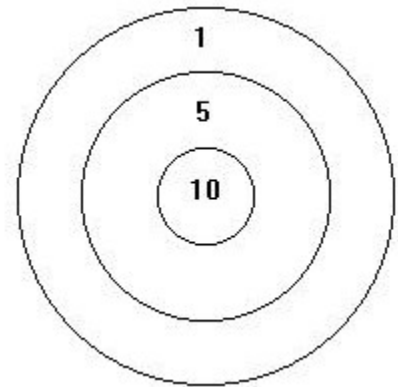
- Does your answer make sense? Did you answer all of the questions?
- What did you learn by doing this?
- Could you have done this problem another way – maybe even an easier way?

## Problem Solving Strategies – Sample Problems for Students to Apply Different Heuristics

Ex. 1 Copy the figure below and place the digits 1, 2, 3, 4, and 5 in these circles so that the sums across (horizontally) and down (vertically) are the same. Is there more than one solution?



Ex. 2 Three darts hit this dart board and each scores a 1, 5, or 10. The total score is the sum of the scores for the three darts. There could be three 1's, two 1's and 5, one 5 and two 10's, And so on. How many different possible total scores could a person get with three darts?



Ex. 3 In a stock car race, the first five finishers in some order were a Ford, a Pontiac, a Chevrolet, a Buick, and a Dodge.

- The Ford finished seven seconds before the Chevrolet.
- The Pontiac finished six seconds after the Buick.
- The Dodge finished eight seconds after the Buick.
- The Chevrolet finished two seconds before the Pontiac.

In what order did the cars finish the race? What strategy did you use?

Ex. 4 South Point Amusement Park has a special package for large groups to attend their amusement park: a flat fee of \$20 and \$6 per person. If a club has \$100 to spend on admission, what is the most number of people who can attend?

Ex. 5 Continue these numerical sequences. Copy the problem and fill in the next three blanks in each part.

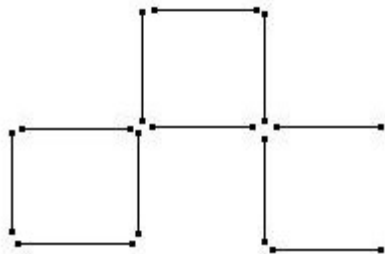
1, 4, 7, 10, 13, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

19, 20, 22, 25, 29, \_\_\_\_\_, \_\_\_\_\_.

2, 6, 18, 54, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

Ex. 6 The houses on Main Street are numbered consecutively from 1 to 150. How many house numbers contain at least one digit 7?

Ex. 7 The figure below shows twelve toothpicks arranged to form three squares. How can you form five squares by moving only three toothpicks?



Ex. 8 Three apples and two pears cost 78 cents. But two apples and three pears cost 82 cents. What is the total cost of one apple and one pear?

Ex. 9 Show how to draw four line segments through the nine dots shown below without lifting your pencil from the paper.



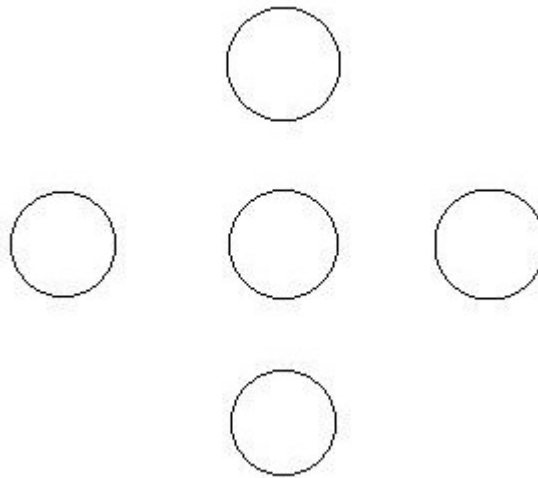
Ex. 10 Two apples weigh the same as a banana and a cherry. A banana weighs the same as nine cherries. How many cherries weigh the same as one apple?

Reardon, T. Teaching Problem Solving Strategies in the 5-12 Curriculum. Retrieved from <http://www.pdfdrive.net/students-problems-using-problem-solving-strategies-pss-1-e8474067.html>.

## More Problem Solving Strategy – Follow-up Problems

### Guess and Check

Put the numbers 2, 3, 4, 5, and 6 in the circles to make the sum across and the sum down equal to 12. Are other solutions possible? List at least two, if possible.



**SOLUTION:** One possibility  
Other solutions possible.  
Have students suggest those.

|   |   |   |
|---|---|---|
|   | 2 |   |
| 3 | 4 | 5 |
|   | 6 |   |

### Make an Organized List

List the 4-digit numbers that can be written using each of 1, 3, 5, and 7 once and only once. Which strategy did you use?

**SOLUTION:**

|      |      |      |      |      |      |
|------|------|------|------|------|------|
| 1357 | 1735 | 3517 | 5137 | 5713 | 7315 |
| 1375 | 1753 | 3571 | 5173 | 5731 | 7351 |
| 1537 | 3157 | 3715 | 5317 | 7135 | 7513 |
| 1573 | 3175 | 3751 | 5371 | 7153 | 7531 |

24 possible 4-digit numbers.

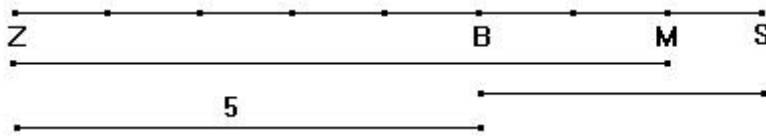
## Draw a Diagram

Four friends ran a race:

- Matt finished seven seconds ahead of Ziggy.
- Bailey finished three seconds behind Sam.
- Ziggy finished five seconds behind Bailey.

In what order did the friends finish the race?

**SOLUTION:**



The order was: Sam, Matt, Bailey, and Ziggy.

## Make a Table

Stacey had 32 coins in a jar. Some of the coins were nickels, the others were dimes. The total value of the coins was \$2.80. Find out how many of each coin there were in the jar. What problem solving strategy did you use?

**SOLUTION:** 8 nickels, 24 dimes

## Look for a Pattern

Copy and continue the numerical sequences:

3, 6, 9, 12, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

27, 23, 19, 15, 11, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

1, 4, 9, 16, 25, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

2, 3, 5, 7, 11, 13, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

**SOLUTION:**

3, 6, 9, 12, 15, 18, 21

27, 23, 19, 15, 11, 7, 3, -1

4, 9, 16, 25, 36, 49, 64

2, 3, 5, 7, 11, 13, 17, 19, 23

multiples of three

subtract 4 from the previous term 1,

perfect squares

prime numbers

### Solve a Simpler Problem

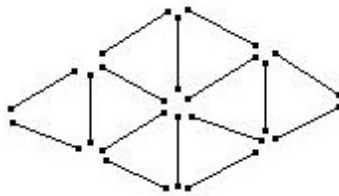
The houses on Market Street are numbered consecutively from 1 to 150. How many house numbers contain at least one digit 4?

#### SOLUTION:

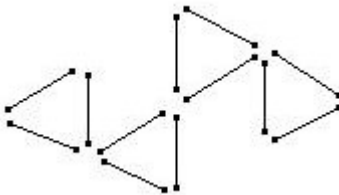
33 house numbers have at least one digit 4

### Act it Out or Use Objects

Sixteen toothpicks are arranged as shown. Remove four toothpicks so that only four congruent triangles remain.



#### SOLUTION:



### Work Backwards

I went into a store and spent half of my money and then \$20 more. I went into a second store and spent half of my money and then \$20 more. Then I had no money left. How much money did I have when I went into the first store?

**SOLUTION:** \$120 to begin with.

### Use Logical Reasoning

Five oranges and a banana cost 87 cents. An orange and five bananas cost 99 cents. What is the total cost of two oranges and two bananas?

**SOLUTION:** 62 cents for two oranges and two bananas

### **Brainstorm and Write an Equation**

Three pears weigh the same as a quince. A quince weighs as much as eighteen raspberries. How many raspberries weigh the same as a pear?

**SOLUTION:** Six raspberries weigh the same as one pear.

Reardon, T. Teaching Problem Solving Strategies in the 5-12 Curriculum. Retrieved from <http://www.pdfdrive.net/students-problems-using-problem-solving-strategies-pss-1-e8474067.html>



## Online Resources

### Mathematical Reasoning

**Annenberg Learner.** Courses of study in such areas as algebra, geometry, and real-world mathematics. The Annenberg Foundation provides numerous professional development activities or just the opportunity to review information in specific areas of study. <http://www.learner.org/index.html>

**Common Core Conversation.** Links to math sites for use with all levels of mathematical standards. <http://www.commoncoreconversation.com/math-resources.html#sthash.Dznxgkbn.dpbs>

**Florida IPDAE.** Lesson plans for both ABE and GED®-level mathematics developed by Florida adult educators. <http://www.floridaipdae.org>

**Free Resources for Educational Excellence.** Teaching and learning resources from a variety of federal agencies. This portal provides access to free resources. <http://free.ed.gov/index.cfm>

**Get the Math.** How algebra is used in real-world situations. <http://www.thirteen.org/get-the-math/>

**Illustrations.** Great lesson plans for all areas of mathematics at all levels from the National Council of Teachers of Mathematics (NCTM). <http://illustrations.nctm.org>

**Inside Mathematics.** A professional resource for educators, including classroom examples of innovative teaching methods and insights into student learning. <http://insidemathematics.org/index.php/home>

**Key Elements to Algebra Success** 46 lessons, homework assignments, and videos. <http://ntnmath.keasmath.com/>

**Khan Academy.** A library of over 2,600 videos covering everything from arithmetic to physics, finance, and history and 211 practice exercises. <http://www.khanacademy.org/>

**The Math Dude.** A full video curriculum for the basics of algebra. [http://www.montgomeryschoolsmd.org/departments/itv/MathDude/MD\\_Downloads.shtm](http://www.montgomeryschoolsmd.org/departments/itv/MathDude/MD_Downloads.shtm)

**Math in the News. Media4Math.** This site provides you with information/articles of how math is used in the real world. <http://www.media4math.com/MathInTheNews.asp>

**Media4Math.** This site provides you with information/articles of how math is used in the real world. <http://www.media4math.com/MathInTheNews.asp>

**Math Planet.** Math Planet is a dedicated web site to the advancement of mathematics. [http://library.thinkquest.org/16284/index\\_s.htm](http://library.thinkquest.org/16284/index_s.htm)

**Online Resources for Teaching and Strengthening Fundamental, Quantitative, Mathematical, and Statistical Skills. NICHE.** A wide array of resources for the different types of mathematical skills. [http://serc.carleton.edu/NICHE/teaching\\_materials\\_gr.html#partone](http://serc.carleton.edu/NICHE/teaching_materials_gr.html#partone)

**National Library of Virtual Manipulatives for Math** - All types of virtual manipulatives or can be purchased as a DVD. This is a great site for students who need to see the “why” of math. <http://nlvm.usu.edu/en/nav/index.html>

**Ohio Resource Center.** Resources, questions, brainteasers – lots of activities for the classroom. [www.ohiorc.org/adilit](http://www.ohiorc.org/adilit)

**PBS Teacher Source.** Lesson plans and lots of activities are included in the teacher section of PBS. <http://www.pbs.org/teachers>

**Real-World Math.** Ideas for how math is used in today’s world. <http://www.realworldmath.org/>

**Teacher Guide for the TI-30XS MultiView™ Calculator** – A guide to assist you in using the new calculator, along with a variety of lesson plans for the classroom. [http://education.ti.com/en/us/guidebook/details/en/62522EB25D284112819FDB8A46F90740/30x\\_mv\\_tg](http://education.ti.com/en/us/guidebook/details/en/62522EB25D284112819FDB8A46F90740/30x_mv_tg)

<http://education.ti.com/calculators/downloads/US/Activities/Search/Subject?s=5022&d=1009>

**TES.** With more than 2.3 million registered online users in over 270 countries and territories, TES provides a wealth of free resources in all academic areas. <http://www.tes.co.uk/>

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# Tips for Attaining High School Equivalency on the GED® Mathematical Reasoning Test

The GED® Mathematical Reasoning test assesses a wide range of math skills and concepts because the test measures both high school equivalency and readiness for college and career. Achieving the level of performance required for high school equivalency does not necessarily require a student to master the entire range of content covered on the test. This resource provides tips that can help instructors to focus on the math skills and performance benchmarks most critical in helping students attain the high school equivalency performance level in math.

## CONSISTENCY

Increase students' consistency in performing math skills. In many cases, the difference between *passing* and *not passing* is that passers perform more consistently, demonstrating deeper understanding of skills they already possess.

## FLUENCY

Increase students' fluency in the basic mathematical operations (e.g., addition, subtraction, multiplication, division, roots, and exponents), mathematical properties (e.g., commutative, associative, distributive), the order of operations (PEMDAS), and the concept of "absolute value."

## NUMBER SENSE

Improve students' general "number sense"

- Students should have a good understanding of the relative size of numbers, helping them to judge the reasonableness of proposed solutions.
- "Number sense" includes being able to order different values on a number line, even if those values are expressed differently (for example, as decimals, fractions, or mixed numbers).

## **MEASUREMENT WITH GEOMETRIC FIGURES**

Increase students' confidence in working with measurement of geometric figures to compute perimeter, area, volume and surface area. Key skills include

- Solving problems with different unknowns. For example, students should be able to compute area given the length and width of a geometric figure and also to compute length given the area and width of a figure.
- Looking at realistic situations and seeing how mathematics can be used to represent those situations. For example, determining the cost of carpeting an irregularly-shaped room involves determining the area and multiplying that by the cost of the carpet per unit. This type of problem also might involve making changes to the units of measurement involved (for example, the room measurements might be expressed in square feet but the carpet might be priced by the square yard).

## **WORKING IN COORDINATE PLANE**

Help students develop strong skills in working in the coordinate plane. Key skills include

- Identifying the elements of the coordinate plane (axes, independent and dependent variables, ordered pairs, etc.).
- Writing the equation of a straight line and going back-and-forth between different equation formats (e.g., point-slope form, slope intercept form, etc.).
- Interpreting the meaning of different elements of the equation of a line (e.g., what does the slope represent? What does the y-intercept represent?).
- Looking at realistic situations and determining how the equation of a line represents those situations.

## **INTERPRETING GRAPHICS**

Help students interpret various tables, charts, and graphs, to explain what they mean, and to move between different ways of expressing the same data (for example, in text, or in a table or in a graph).

## **MEAN, MEDIAN, & MODE**

Ensure that students are able to demonstrate basic skills with measures of central tendency, including computing and interpreting mean, median, and mode.

## Tips for the Calculator-Prohibited Section of the GED® Mathematical Reasoning Test

Non-calculator questions make up about 12% of the points on the GED® Mathematical Reasoning Test and assess foundational arithmetic skills including

- The four basic operations (addition, subtraction, multiplication, and division)
- Exponents and roots,
- Order of operations,
- Scientific notation, and
- Basic number sense.

The following examples of calculator-prohibited questions address the skills students need in each of the identified GED® test assessment target/indicator areas. For multiple choice questions, the correct answer is indicated by an asterisk (\*).

### Q.1.a: Order fractions and decimals, including on a number line.

A list of numbers is shown.

$$\frac{3}{4}, 0.6, \frac{5}{16}, 0.15, \frac{3}{8}$$

Which list shows the numbers in order from least to greatest?

**A\***      $0.15, \frac{5}{16}, \frac{3}{8}, 0.6, \frac{3}{4}$

**B**      $\frac{5}{16}, 0.15, \frac{3}{8}, 0.6, \frac{3}{4}$

**C**      $0.6, 0.15, \frac{3}{4}, \frac{3}{8}, \frac{5}{16}$

**D**      $0.15, 0.6, \frac{5}{16}, \frac{3}{8}, \frac{3}{4}$

### Q.1.d: Identify absolute value of a rational number as its distance from 0 on the number line and determine the distance between two rational numbers on the number line, including using the absolute value of their difference.

The numbers -8 and -3 are plotted on a number line. What is the distance, in units, between the two points?

**A** -11     **B** -5     **C\*** 5     **D** 11

**Q.2.a: Perform addition, subtraction, multiplication, and division on rational numbers.**

Multiply.

$$2.25 \times 1.6$$

- A 0.1675
- B 0.36
- C 1.675
- D\* 3.6

**Q.2.b: perform computations and write numerical expressions with squares and square roots of positive, rational numbers.**

An expression is shown.

$$\sqrt{15} \cdot \sqrt{12}$$

Simplify the expression completely. Leave your answer in radical form. Type your answer in the box. Use only numbers and symbols in your answer. (NOTE: Click the symbol selector when you need to enter the radical sign.)

Correct answer:  $6\sqrt{5}$

**Q.2.d: Determine when a numerical expression is undefined.**

What value of  $x$  makes the expression  $\frac{1}{2x}$  undefined?

- A -2
- B -1
- C\* 0
- D  $\frac{1}{2}$