Introduction to Higher-Order Algebra for Level 1 and Level 2 Students

A Workshop by GED Testing Service®

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Session Objectives

- Determine the big ideas of algebra
- Discuss the importance of teaching the basics of inequalities and functions (two High Impact Indicators)
- Connect inequalities and functions to real-world situations
- Share resources

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The Magic of Algebra

- Think of a number between 1 and 100.
- Multiply your number by 4.
- Add 12.
- Multiply this number by 2.
- Add 16.
- Divide this number by 8.
- Subtract your original number.

Your new number is 5!

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Can You Show that Algebraically?

Think of a number between 1 & 100.	Let's represent this number as n.
Multiply your number by 4.	We can show this as 4n (4 times n).
Add 12.	4n + 12
Multiply this number by 2.	2 (4n + 12) = 8n + 24 (using the Distributive Property)
Add 16.	(8n + 24) + 16 = 8n + (24 + 16) = 8n + 40 (using the Associative Property)
Divide this number by 8.	$\frac{8n+40}{8} = \frac{8n}{8} + \frac{40}{8}$ = n + 5
Subtract your original number.	n + 5 – n = 0 + 5 (using the Commutative and Associative Properties)
Your number is 5!	0 + 5 = 5

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The Magic of Algebra (What's the reason?)

Think of any number.

- Multiply it by 2.
- Add 4.
- Multiply by 3.
- Divide by 6.
- Subtract the number with which you started.

You got 2!

Explain with algebra why this works.

6

The answer is . . .

Start with the expression that describes the operations to be performed on your chosen number, *x*:

$$\frac{(2x + 4) \cdot 3}{6} - x$$

and simplify the expression. You'll end up with 2, regardless of the value of *x*.

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Why Use Magic Tricks or Puzzles?

- They are
  - Fun
  - Non-threatening
  - Motivational
  - Engaging
- Students begin to use algebraic thinking without knowing that is what they are doing.

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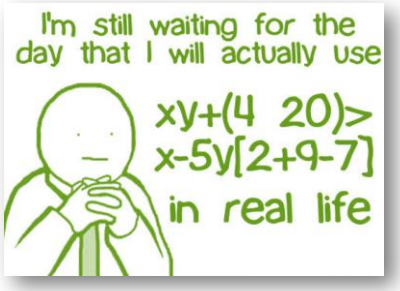
Some Big Ideas in Algebra



- Variable
- Symbolic Notation
- Equality
- Ratio and Proportion
- Pattern Generalization
- Equations and Inequalities
- Multiple Representations of Functions

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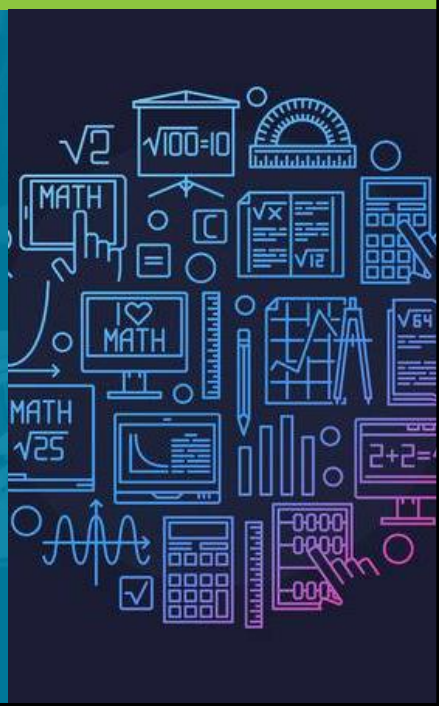
Holding Algebra in Your Hands

Starting with the concrete by using algebra tiles



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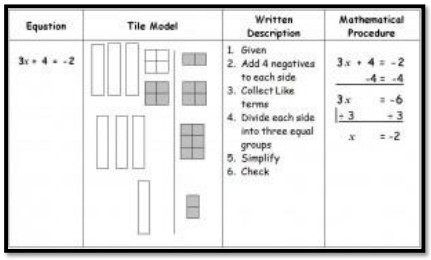
When teaching algebraic concepts, model using multiple representations

- Start with the concrete
- Represent problems using symbols, expressions, and equations, tables, and graphs
- Model real-world situations
- Complete problems different ways (flexibility in problem solving)



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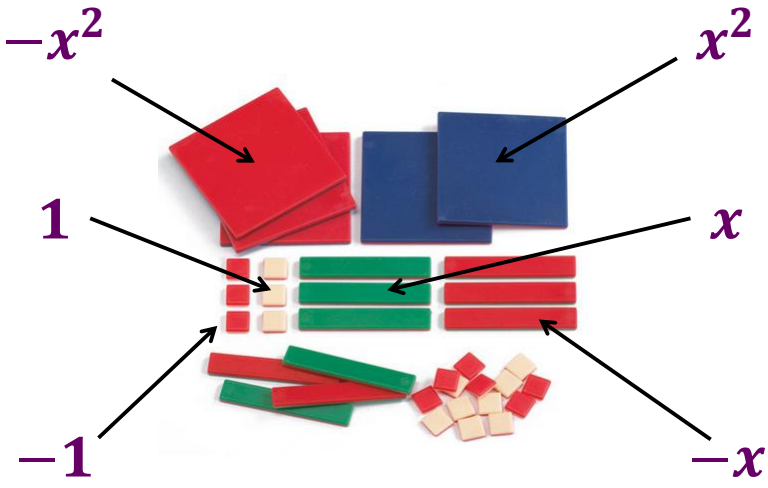
Introduction to Algebra Tiles



Remember, they could be called x, y, b, t, etc.

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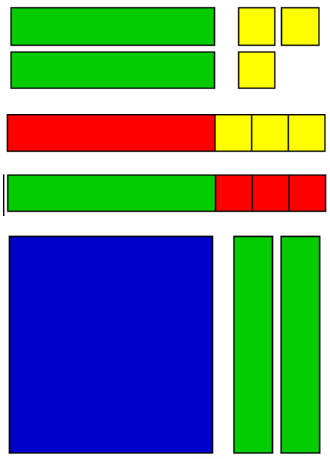
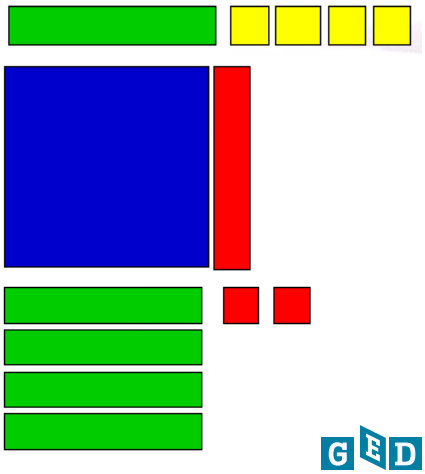
What's My Polynomial?





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Big Ideas Using Algebra Tiles

- Adding and Subtracting Integers; Zero Principle
- Modeling Linear Expressions
- Solving Linear Equations
- Simplifying Polynomials
- Solving Equations for Unknown Variable
- Multiplying and Dividing Polynomials
- Factoring Trinomials
- Completing the Square
- Investigations

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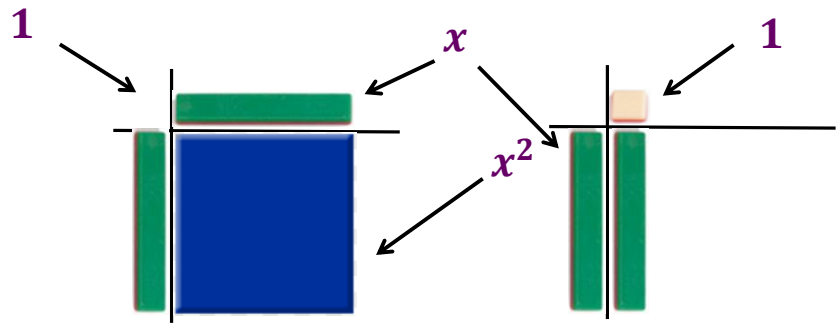
Look at the Relationships among the Tiles



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Let's Use Algebra Tiles to Multiply Polynomials

Problem: (x + 2)(x + 3) = ?

Step 1: Gather all the tiles you need for each term.

x + 2 

x + 3 

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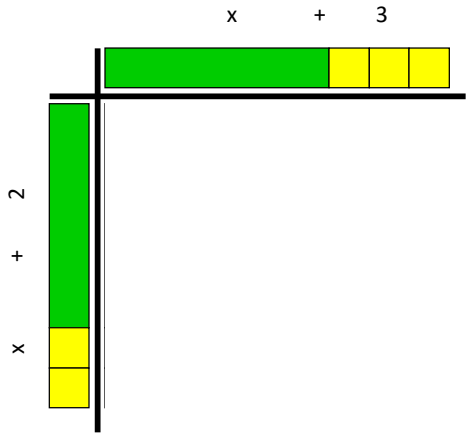
Multiplying Polynomials

$(x + 2)(x + 3) = ?$

Step 2: Lay out the tiles on opposite axes of the algebra tile mat as shown below.



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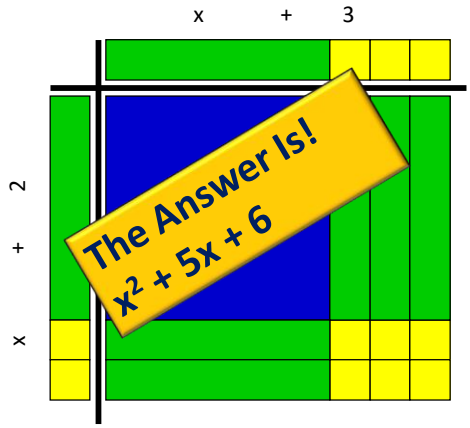
Multiplying Polynomials

$(x + 2)(x + 3) = ?$

Step 3: Using other tiles, form a perfect rectangle using the tiles on axes of the product mat as the indicators for the dimensions of the rectangle as shown.



Notice that no other combination of tiles would make a perfect rectangle matching the dimension of tiles on the axes of the product mat.

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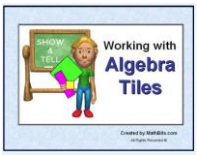
Resources



- Working With Algebra Tiles – MathBits
  http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles.htm
- Factoring Polynomials Using Algebra Tiles - Del Mar College
  http://dmc122011.delmar.edu/math/MLC/QEPMathSeminars/FactoringTrinomialsAlgebraTilesStudentActivity.pdf
- Multiplying Polynomials Using Algebra Tiles – Virginia Dept. of Education
  http://www.doe.virginia.gov/testing/solsearch/sol/math/A/mess_a-2b_2.pdf

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Algebra Tile Apps

- Illuminations (National Council for Teachers of Math)
  http://illuminations.nctm.org/activity.aspx?id=3482

- Michigan Virtual University
  http://media.mivu.org/mvu_pd/a4a/homework/index.html

- National Library of Virtual Manipulatives
  http://nlvm.usu.edu/en/nav/vlibrary.html

Don't forget phone apps!

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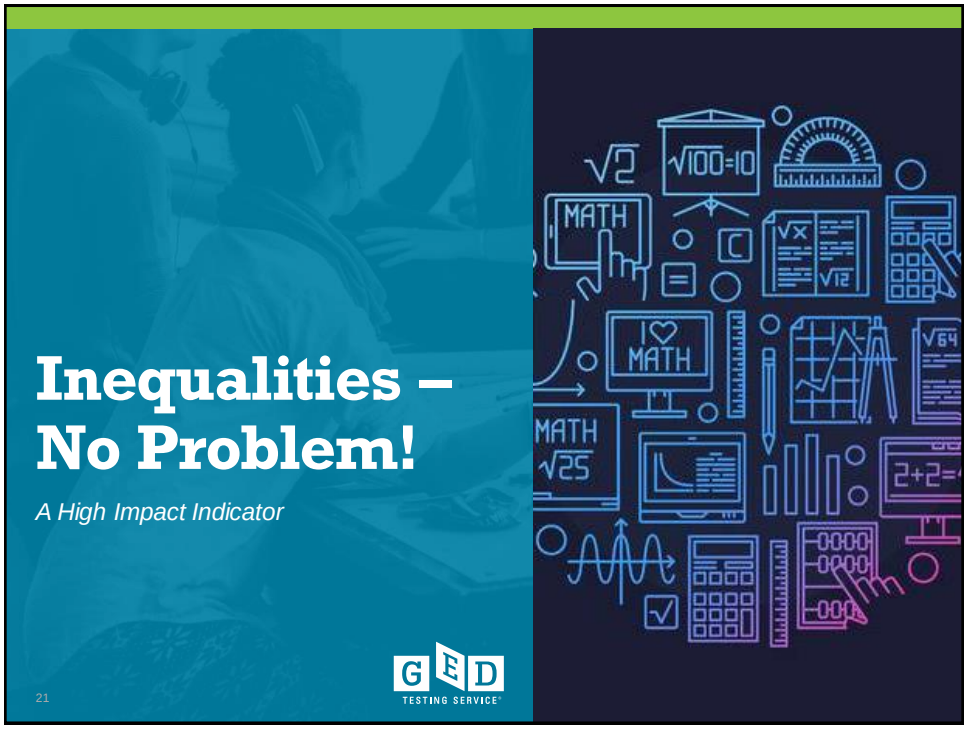

Focusing on High Impact Indicators - Inequalities

A.3 Write, manipulate, solve, and graph linear inequalities

- **A.3.a** Solve linear inequalities in one variable with rational number coefficients.
- **A.3.b** Identify or graph the solution to a one variable linear inequality on a number line.
- **A.3.c** Solve real-world problems involving inequalities.
- **A.3.d** Write linear inequalities in one variable to represent context.

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Can your students . . .

Solve inequalities in one variable, using the standard algorithms?

Solve a one-variable inequality and identify or create a graph on the number line of the solution?

Analyze the relationship between quantities in a real-world problem, and then create an inequality to model the problem situation?

Analyze the relationship between quantities in a real-world problem, and then solve the problem through algebraic reasoning?

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What is an inequality?

An inequality is a mathematical sentence that uses symbols such as <, ≤, >, or ≥ to compare two quantities.

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Inequalities Are Everywhere

Situation	Mathematical Inequality
Speed limit	Legal speed on the highway ≤ 65 miles per hour
Credit card	Monthly payment ≥ 10% of your balance in that billing cycle
Text messaging	Allowable number of text messages per month ≤ 250
Travel time	Time needed to drive from home to school/work ≥ 18 minutes

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Recognize the Symbols and the Vocabulary

Phrase	Inequality
"a is more than b"	a > b
"a is at least b"	a ≥ b
"a is less than b"	a < b
"a is at most b;" or "a is no more than b"	a ≤ b

Inequality tells what is "allowable" or "possible." An inequality places conditions on the value of the variable.

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Recognize the Symbols and the Vocabulary

Term	Inequality
Coefficient	4a>8
Boundary Point	A solution that makes the inequality true
Solution Set	The range of values that make the inequality true
Inclusive	a ≤ 6 
Exclusive	a < 6 

An inequality tells what is “allowable” or “possible.”

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Rules for Solving Inequalities

1. Make the same changes to both sides of the inequality
2. Isolate the variable
3. Combine like terms
4. Use the inverse operation to remove clutter from the variable
5. If your inverse operation is multiplication or division by a negative number, reverse the inequality sign

< becomes >
> becomes <
≤ becomes ≥
≥ becomes ≤

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Properties of Inequalities

Addition and Subtraction

If a > b, then a + c > b + c

If a > b, then a - c > b – c

Real-life situation

Becky is older than Janet: b > j

Add 10 years: b + 10 > j + 10

Subtract 10 years: b – 10 > j - 10

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Properties of Inequalities

Multiplication and Division

If a > b, then ac > bc, if c > 0

If a < b, then ac < bc, if c <0

Real-life situation

Becky is older than Janet: b > j

When they are twice their current age:

b(2) > j(2)

When they were half the age they are now:

$\frac{b}{2} > \frac{j}{2}$

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But…there is one exception

$-3n > 12$ If you divide or multiply by a negative number

$\frac{-3n}{-3} > \frac{12}{-3}$ reverse the inequality symbol

$n < -4$

Solution: all numbers less than -4

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Testing the Property

$3 > 2$

Multiply by -1

(-1)(3) > 2 (-1)

-3 > -2 **FALSE**

-3 < -2 **TRUE**

Multiplying by a negative flipped the inequality sign from "greater than" to "less than."

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Solve the Inequality

$$4 + x < 12$$

$4 + \boxed{x} < 12$ (draw wall down inequality)

$4 + x < 12$ (box in variable)

$4 + x < 12$ (minus 4 both sides)

$-4 \qquad -4$

$x < 8$

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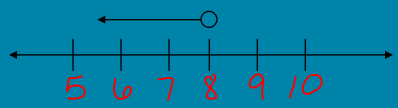
Graph the Solution

$$x < 8$$

1. Draw a number line. Just need a few numbers on either side of the solution number.



2. Decide if open circle or closed circle. Place it above the solution number.
3. Determine which way your arrow goes by substituting a number in for the variable to make the statement true. Then draw the arrow pointing in that direction.

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Solve the Inequality

$$48 \geq 8m$$

$\frac{48}{8} \geq \frac{8m}{8}$ (draw wall, box variable)

(divide 8 both sides)

$$6 \geq m$$

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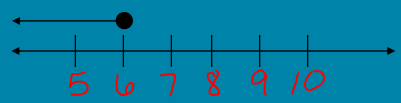
Graph the Solution

$$6 \geq m$$

1. Draw a number line. Just need a few numbers on either side of the solution number.



2. Decide if open circle or closed circle. Place it above the solution number.
3. Determine which way your arrow goes by substituting a number in for the variable to make the statement true. Then draw the arrow pointing in that direction.

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Your Turn!

Coach told us we needed to complete more than 18 repetitions.

Which one is correct?

1. $N \geq 18$
2. $N \leq 18$
3. $N > 18$
4. $N < 18$

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Your Turn!

Children under 12 pay a discounted price at the movie theatre.

Which one is correct?

1. $N \geq 12$
2. $N \leq 12$
3. $N > 12$
4. $N < 12$

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Your Turn!

Marie has purchased party supplies for a maximum of 12 people.

Which one is correct?

1. $N \geq 12$
2. $N \leq 12$
3. $N > 12$
4. $N < 12$

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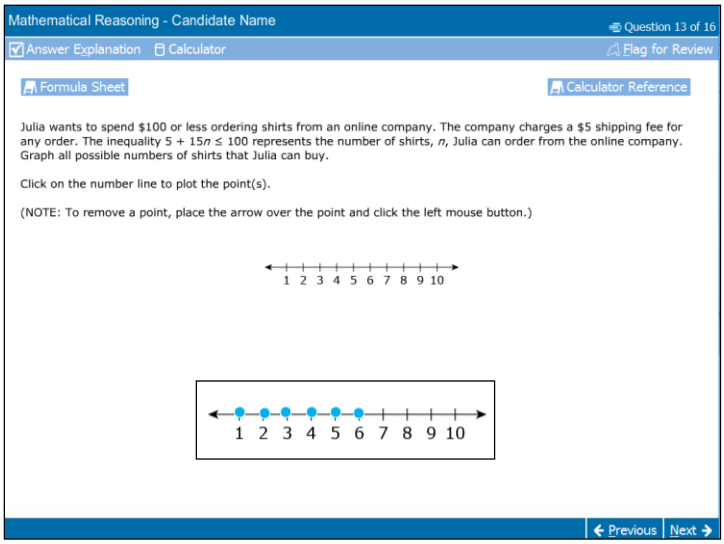
Sample Question from GEDTS



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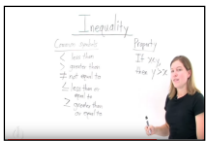
Resources

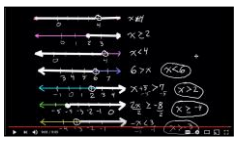
One-Variable Inequalities – Khan Academy

https://www.khanacademy.org/math/algebra/one-variable-linear-inequalities



Virtual Nerds: What is an Inequality?

https://www.youtube.com/watch?v=wcBwdz-ZBaM



Very Basics of Graphing Inequalities (on a number line)

https://www.youtube.com/watch?v=nif2PKA9bXA

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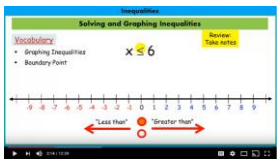
Resources

Math is Fun – Solving Inequalities

http://www.mathsisfun.com/algebra/inequality-solving.html



Solving and Graphing Inequalities (Excellent!)

https://www.youtube.com/watch?v=EE2qWIyjKD0

Math Dude Unit 1-4 – Solving Inequalities

https://www.youtube.com/watch?v=8hhewFQ_K0w

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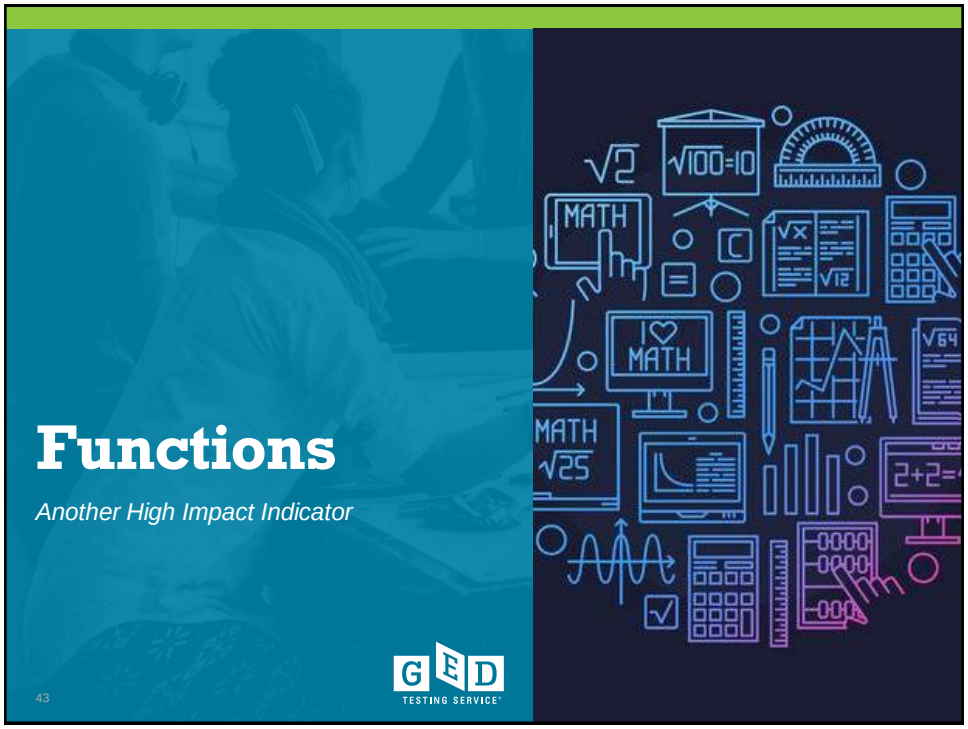

Focusing on High Impact Indicators - Functions

A.7 Compare, represent, and evaluate functions.

- **A.7.a** Compare two different proportional relationships represented in different ways.
- **A.7.b** Represent or identify a function in a table or graph as having exactly one output (one element in the range) for each input (each element in the domain).
- **A.7.c** Evaluate linear and quadratic functions for values in their domain when represented using function notation.
- **A.7.d** Compare properties of two linear or quadratic functions each represented in a different way (algebraically, numerically in tables, graphically or by verbal descriptions).

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Can your students . . .

Identify functions and non-functions displayed in graphs and tables, and create functions (graphs/tables)?

Substitute values for variables in functions and evaluate the resulting numerical expressions?

Convert functional representations from one from to another, and compare properties of the functions?

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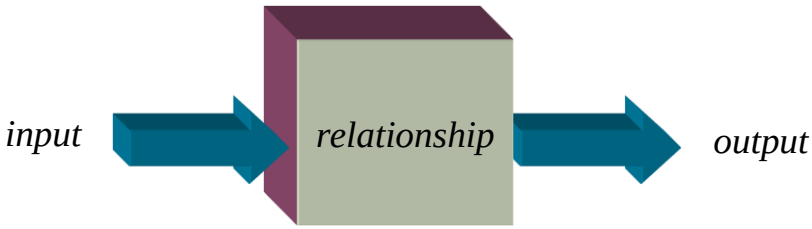
What is a function?

A **function** is a special relationship between a set of inputs and a set of permissible outputs. It is a useful mathematical tool.



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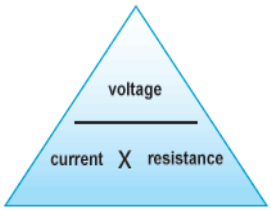
Functions in Daily Life



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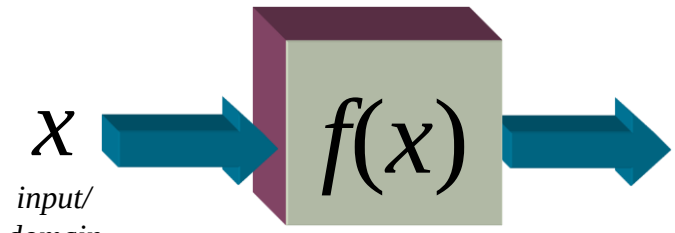
What is a function? Looking Closer

A **function** is a relation in which each element of the domain is paired with *exactly one* element of the range. Another way of saying it is that there is one and only one output (y) with each input (x).



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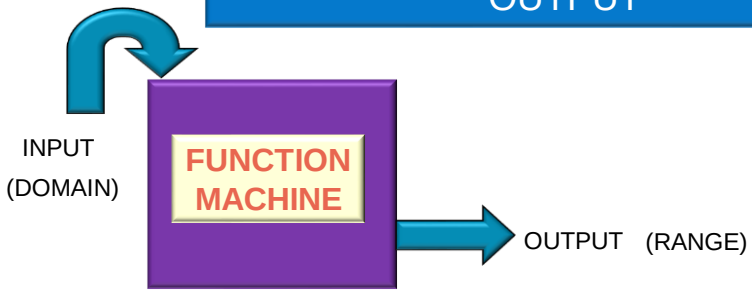
In order for a relationship to be a function . . .

EVERY INPUT MUST HAVE AN OUTPUT

TWO DIFFERENT INPUTS CAN HAVE THE SAME OUTPUT

ONE INPUT CAN HAVE ONLY ONE OUTPUT



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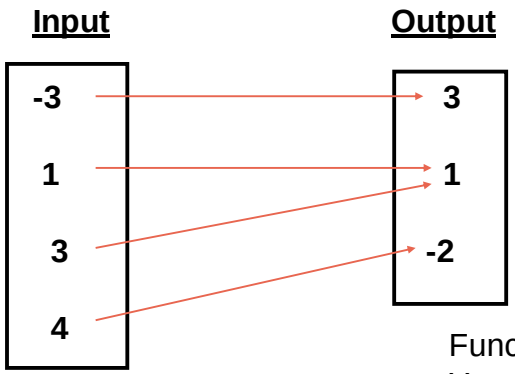
Is it a function?



Domain (input) = {-3, 1, 3, 4}
Range (output) = {3,1,-2}

Function?
Yes: each input is mapped onto exactly one output

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Real World – Is It a Function?

People and Social Security Numbers

Domain	All people with a valid social security number
Range	All valid social security numbers
Is it a function?	Yes

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Real World – Is It a Function?

People and Phone Numbers

Domain	All people who have a phone
Range	Phone numbers of all people who have a phone
Is it a function?	No

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Show Functions Four Ways

x	y
-4	-11
-2	-7
0	-3
2	1
4	5

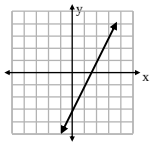
Table

$y = 2x - 3$

Equation

y is 3 less than twice a number x

Written Description



Graph

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Vocabulary

x	*y*
Input	Output
Domain	Range
Independent Variable	Dependent Variable
x	f(*x*)

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Is it a function?
Creating Input/Output Tables

{(-5,3), (6,5), (3,2), (1, -3)}

There is only one output for each input. A relationship does exist, so yes, it is a function.

Input x	*Output y*
-5	3
6	5
3	2
1	-3

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Is it a function?
Creating Input/Output Tables

{(4,3), (-2, 10), (4, -6), (10,7)}

There are two inputs that are the same number, but each has a different output. A relationship does not exist, so no, it is not a function.

Input x	*Output y*
4	3
-2	10
4	-6
10	7

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Is it a function?

x	*y*
-10	43
-15	2.5
52	3
33	6
-10	5.5
0	4
53	5

True or False?
Why?

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Is it a function?

Which of the following relations are functions?

R= {(9,10), (-5, -2), (2, -1), (3, -9)}

S= {(6, a), (8, f), (6, b), (-2, p)}

T= {(z, 7), (y, -5), (r, 7) (z, 0), (k, 0)}

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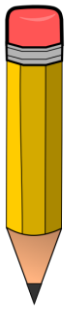
Is it a function?

Vertical Line Test: *a relation is a function if a vertical line drawn through its graph, passes through only one point.*



AKA: ***"The Pencil Test"***

Take a pencil and move it from ***left to right (–x to x)****; if it crosses more than one point, it is not a function.*

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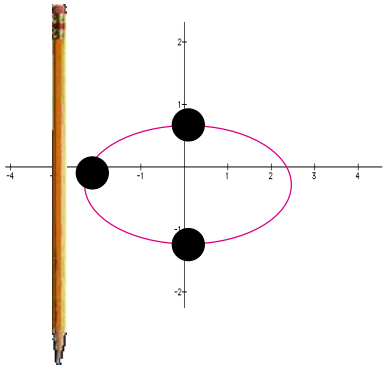
Vertical Line Test



Would this graph be a function?

NO

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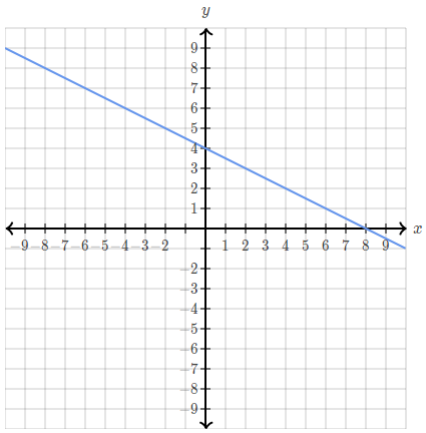
Is it a function?



Yes

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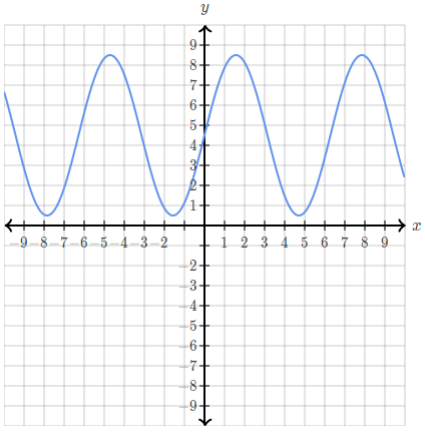
Is it a function?



Yes

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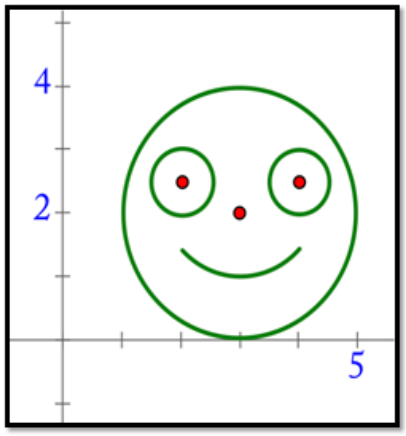
Is it a function?



No

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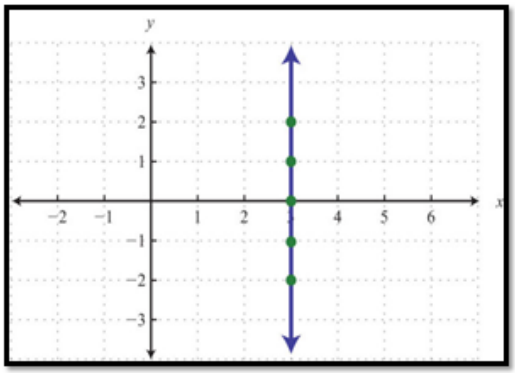
Is it a function?



No

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Back to Real-World Situations

Is it a function?

- The relation of distance and time during a trip.
- The relation of a month to the length of daylight.
- The relation of a person's shoe size to their height.
- The relation of amount of money earned and hours worked.

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Function Notation

$$y = 3x + 2$$

$$f(x) = 3x + 2$$

Name of Function

Input

Output

66

Evaluate the Function

Find f (-2).

To find f(-2) you need to substitute a -2 for every *x* value. Then carefully simplify using the order of operations.

$$f(x) = 2x^2 - 3x + 6$$

$$f(-2) = 2(-2)^2 - 3(-2) + 6$$

$$f(-2) = 2(4) - 3(-2) + 6 = 8 + 6 + 6 = 20$$

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It's Your Turn!

Given f(*x*) = 3x + 20, find f(-4)
= 3(-4) + 20
= -12 + 20
= 8

To find f(-4) you need to substitute a -4 for every *x* value. Then carefully simplify using the order of operations.

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It's Your Turn!

Given that the height of a particular object at time 6 is: $h(t) = 50\,t - 4.9t^2$, find $h(2)$

$h(2) = 50(2) - 4.9(2)^2 = 100 - 19.6 = 80.4$

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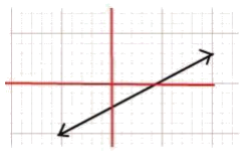
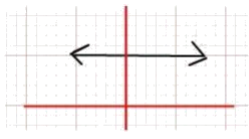
Types of Functions – The Next Step

Linear

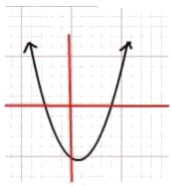


Linear

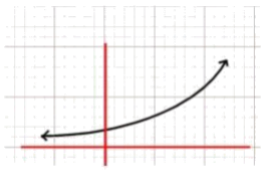


Constant

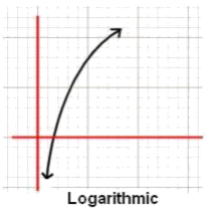
Non-Linear



Quadratic



Exponential



Logarithmic

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A Real-World Linear Function

A lawyer charges a base (one time) fee of $200 and $75 each hour for consulting with her. Calculate the total cost of the lawyer if you consulted with her for one, two, three, four, or five hours.

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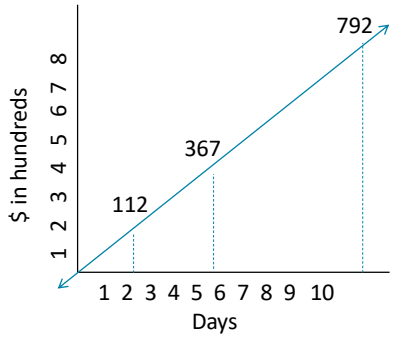
Remember Harley?

a) $m = \frac{367-112}{5-2} = \frac{255}{3} = 85$

b = 367 – 85(5) = -58

b) y = 85x – 58

c) y = 85(10) – 58 = 850 – 58 = $792



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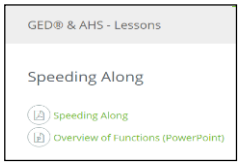
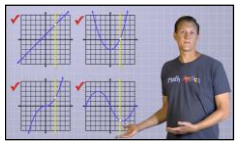
Resources – Beginning Looks





- Using a Lottery to Illustrate Functions - The Teaching Channel https://www.teachingchannel.org/videos/teaching-functions?utm_source=Alpha+List&utm_campaign=17fa2b7690-
- Speeding Along http://www.floridaipdae.org/index.cfm?fuseaction=resources.GEDAHS&cagiid=A37BC967EEFD18737E7AC2AF2D8421DD4A11C694934330A61EB65F4EB10E766B
- What Are Functions? Math Antics https://www.youtube.com/watch?v=52tpYl2tTqk

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Resources

Functions – Khan Academy https://www.khanacademy.org/math/algebra/algebra-functions

What is a function? https://www.youtube.com/watch?v=ryQJa8ybxVY

Math is Fun https://www.mathsisfun.com/sets/function.html

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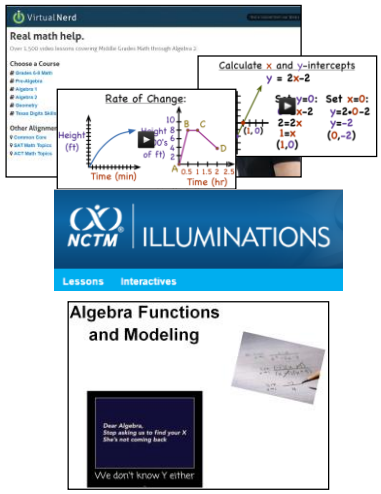
Resources



Virtual Nerd
http://www.virtualnerd.com/

Illuminations
https://illuminations.nctm.org/

Algebraic Functions and Modeling – Steve Schmidt, Appalachian State
https://abspd.appstate.edu/node/385

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Tips for Teaching Inequalities and Functions

- Make it meaningful - start with concrete examples and real-world problems
- Make your thinking processes visible
- Solve the problems many ways
- Show the application
- Provide time for discourse - have students communicate their reasoning
- Ensure time for mastery of the basics

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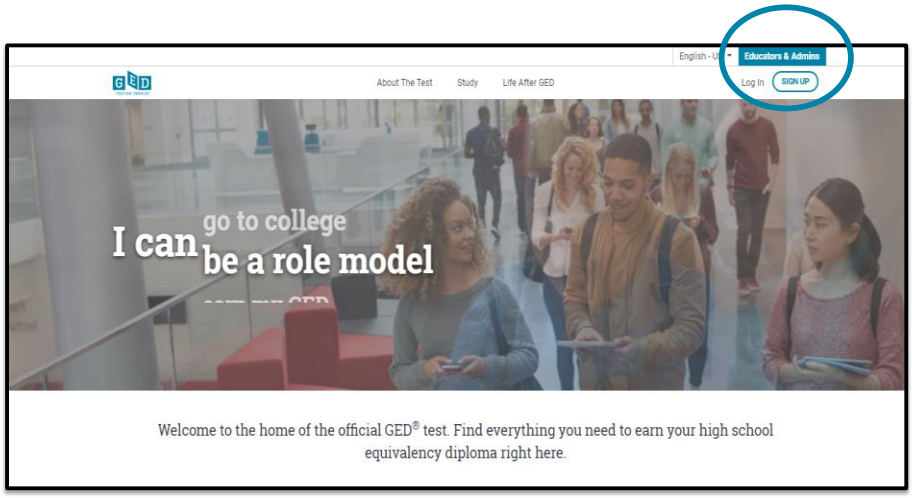
https://ged.com



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