

2023 GED CONFERENCE

Grasping GED® Higher Order Math Concepts for Deeper Understanding

July 18, 2023













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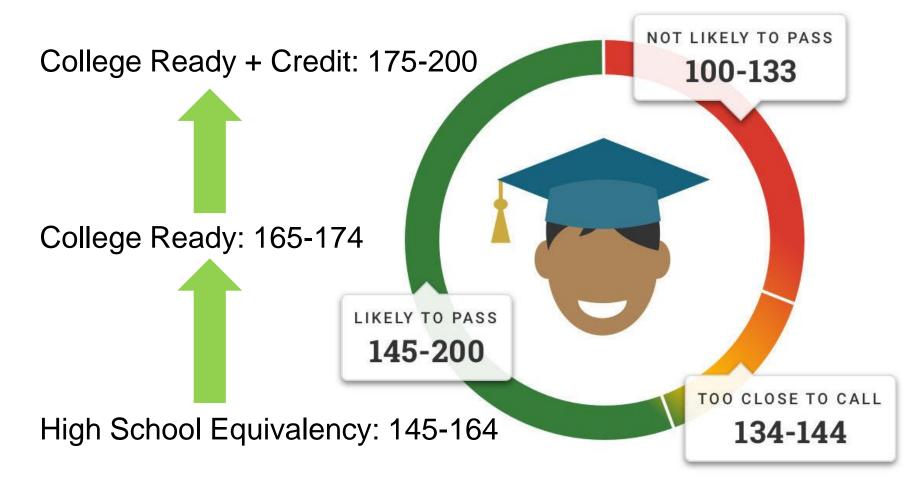
In this session, we will

- Discuss how to push high achieving students to score at a college ready or college ready + credit level on the mathematical reasoning test.
- Focus on critical performance gaps based on field testing data
- Explore strategies, resources and ideas to address these performance gaps.





Three Score Level Indicators on GED Ready®





How do we make high performing GED students perform even better?





- Non-Calculator Items
- 2. Exponents and Roots
- 3. Three-Dimensional Shapes
- 4. Algebraic Computations
- 5. Inequalities
- 6. Slope and Graphing Linear Equations
- 7. Multiple Correct Answers

Tuesdays for Teachers:

GED Knowledge & Skill Gaps - Math

Session 1 - Oct. 26, 2021

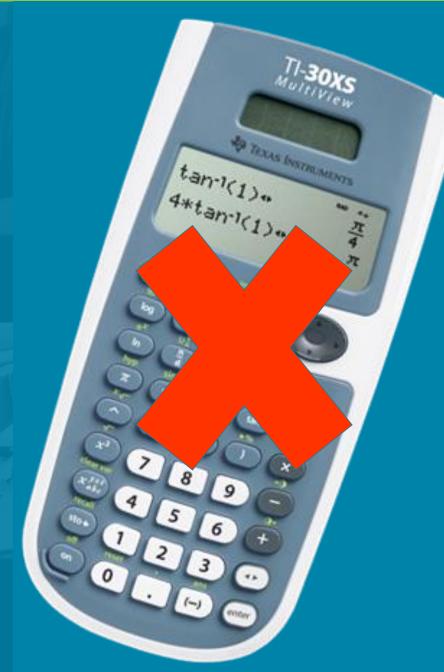
Session 2 - Nov. 16, 2021



Calculator-Prohibited Indicators

Performance Gap 1





Non-Calculator Items

- Ordering Rational Numbers
- 2. Factors and Multiples
- Distance on a Number Line
- 4. Operations on Rational Numbers
- Rules of Exponents
- 6. Squares and Square Roots of Positive Rational Numbers
- 7. Cubes and Cube Roots of Rational Numbers
- Undefined Value Over the Set of Real Numbers







60 Second Challenge: Ordering Rational Numbers



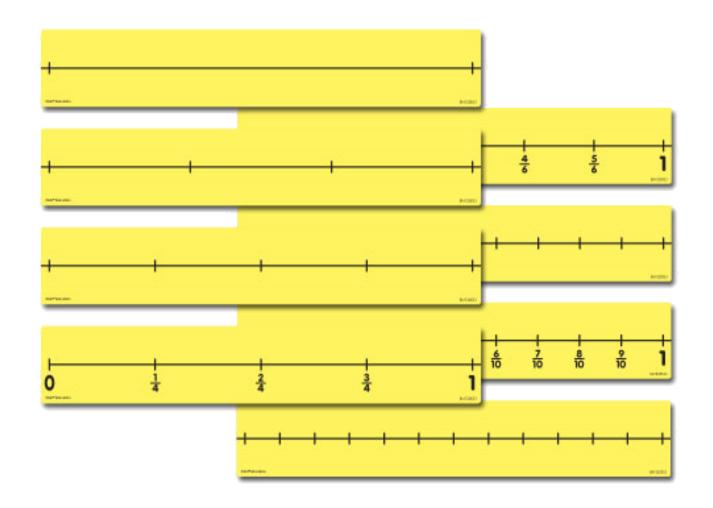
Arrange the given numbers in ascending order.

$$\frac{5}{8}$$
, -3, 0.8314, $\frac{1}{16}$, - π , 0.4823, $\frac{5}{12}$





Let's Try Again

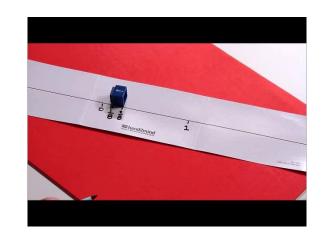




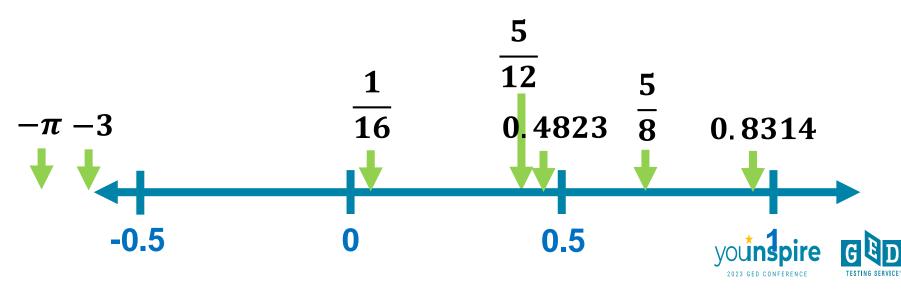


Ordering Rational Numbers: Use a Number Line

$$\frac{5}{8}$$
, -3, 0.8314, $\frac{1}{16}$, - π , 0.4823, $\frac{5}{12}$



- Draw a number line and mark benchmark numbers.
- 2. Plot the easiest given numbers in reference to benchmark numbers.
- 3. Compare and plot the last remaining numbers.



Factors and Multiples

Greatest Common Factor – Used to simplify fractions

Example:
$$\frac{22}{12} = \frac{22 \div 2}{12 \div 2} = \frac{11}{6}$$

• Least Common Multiple (LCM) – Used to add/subtract fractions with unlike denominators.

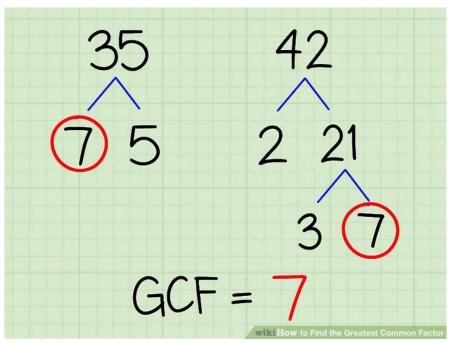
Example:
$$\frac{7}{6} - \frac{1}{4} = \frac{14 - 3}{12} = \frac{11}{12}$$



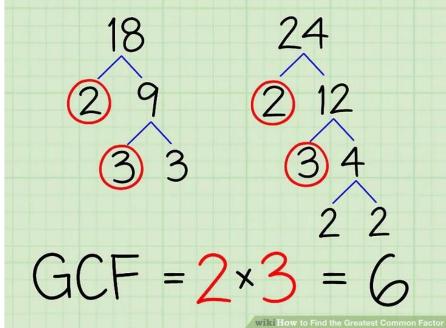


Greatest Common Factor

Example 1: Find the GCF of 35 and 42.



Example 2: Find the GCF of 18 and 24.







Least Common Multiple (LCM)

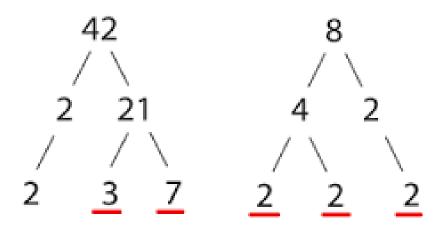
Example 1: Find the LCM of 8, 4, and 6.

Method 1: Listing Multiples

$$8 \rightarrow 8$$
, 16, 24, 32, 40, 48
 $4 \rightarrow 4$, 8, 12, 16, 20, 24, 28, 32
 $6 \rightarrow 6$, 12, 18, 24, 30, 36

LCM = 24

Example 2: Find the LCM that is necessary to perform the indicated operation: $\frac{3}{8} - \frac{1}{42} =$



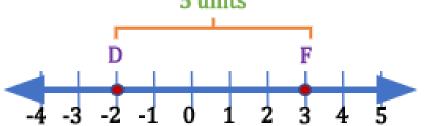


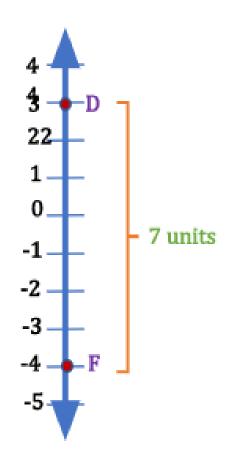


Distance on a Number Line

The distance is how far apart points are on a number line.

5 units





Find the distance between -24 and 13 on a number line.





Distance on a Number Line

The distance between two points A and B on a number line is:

$$= |A - B|$$

Example:

Find the distance between -24 and 13 on a number line.

$$= |-24 - 13|$$

= $|-37|$
= 37

Absolute Value is the distance of a number from zero on a number line.





Operations on Rational Numbers

By MathTricks on Facebook Reels

https://www.facebook.com/reel/657776426 087451/







Exponents and Roots/Radicals

Performance Gap 2



finally found the square root!



Rules of Exponents

Workbook P. 3

Name	Rule	Example
Product	$a^m \cdot a^n = a^{m+n}$	$x^3 \cdot x^4 = x^{3+4} = x^7$
Quotient	$a^m \div a^n = a^{m-n}$	$p^5 \div p^2 = p^{5-2} = p^3$
Power of a Power	$(a^m)^n = a^{mn}$	$(z^3)^2 = z^{3 \cdot 2} = z^6$
Power of a Product	$(ab)^m = a^m b^m$	$(3y)^2 = 3^2y^2 = 9y^2$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$
Zero Exponent	$a^0 = 1$	$x^0 = 1$; $6^0 = 1$; $0^0 = 1$
Negative Exponent	$a^{-m} = \frac{1}{a^m}$	$b^{-3} = \frac{1}{b^3}; 5^{-2} = \frac{1}{5^2}$
Fractional Exponent	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$4^{\frac{3}{2}} = \sqrt[2]{4^3} = \sqrt{64} = 8$

Square and Square Root Tricks (Part 1)

Combining of Similar Radicals

$$a\sqrt{b} + a\sqrt{b} = (a+a)\sqrt{b}$$

$$a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$$

Example 1:
$$2\sqrt{5} + 6\sqrt{5} = (2+6)\sqrt{5} = 8\sqrt{5}$$

Example 2:
$$3\sqrt{2} - 5\sqrt{2} = (3-5)\sqrt{2} = -2\sqrt{2}$$



Square and Square Root Tricks (Part 2)

Splitting Products

$$\sqrt{x^3} = \sqrt{x^2 \cdot x} = \sqrt{x^2} \sqrt{x} = |x| \sqrt{x}$$
 $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = |2| \sqrt{5}$

Splitting Quotients

$$\sqrt{\frac{x^2}{y^2}} = \frac{\sqrt{x^2}}{\sqrt{y^2}} = \frac{x}{y}$$

$$\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{|2|}{|5|}$$



Simplify
$$2\sqrt{2}(2\sqrt{3} + 3\sqrt{3})$$

$$=2\sqrt{2}(5\sqrt{3})=10\sqrt{2\cdot 3}=10\sqrt{6}$$

Simplify $3\sqrt{24x^3}$

$$=3\sqrt{4\cdot 6\cdot x^2\cdot x}=3\cdot 2\cdot x\sqrt{6x}=6x\sqrt{6x}$$

Simplify $\left(-4\sqrt{2}\right)^2$

$$= (-4)^2 \left(\sqrt{2}\right)^2 = 16 \cdot 2 = 32$$

Simplify
$$\sqrt{\frac{12x^2}{4}} = \frac{\sqrt{12x^2}}{\sqrt{4}} = \frac{\sqrt{4 \cdot 3 \cdot x^2}}{2} = \frac{2x\sqrt{3}}{2} = x\sqrt{3}$$

Cube and Cube Root Tricks (Part 1)

Combining of Similar Radicals

$$a\sqrt[3]{b} + a\sqrt[3]{b} = (a+a)\sqrt[3]{b}$$

$$a\sqrt[3]{b} - c\sqrt[3]{b} = (a-c)\sqrt[3]{b}$$

Example 1:
$$2\sqrt[3]{5} + 6\sqrt[3]{5} = (2+6)\sqrt[3]{5} = 8\sqrt[3]{5}$$

Example 2:
$$3\sqrt[3]{2} - 5\sqrt[3]{2} = (3-5)\sqrt[3]{2} = -2\sqrt[3]{2}$$



Cube and Cube Root Tricks (Part 2)

Splitting Products

$$\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = \sqrt[3]{x^3} \cdot \sqrt[3]{x} = x\sqrt[3]{x}$$

$$\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$$

Splitting Quotients

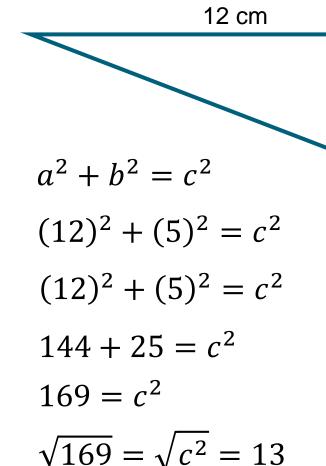
$$\sqrt[3]{\frac{x^3}{y^3}} = \frac{\sqrt[3]{x^3}}{\sqrt[3]{y^3}} = \frac{x}{y}$$

$$\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$$



More Examples: Exponents and Roots

1. Find the length of the hypotenuse of the right triangle.





5 cm

Students must memorize the first 12 perfect squares (1, 4, 9, ..., 144).



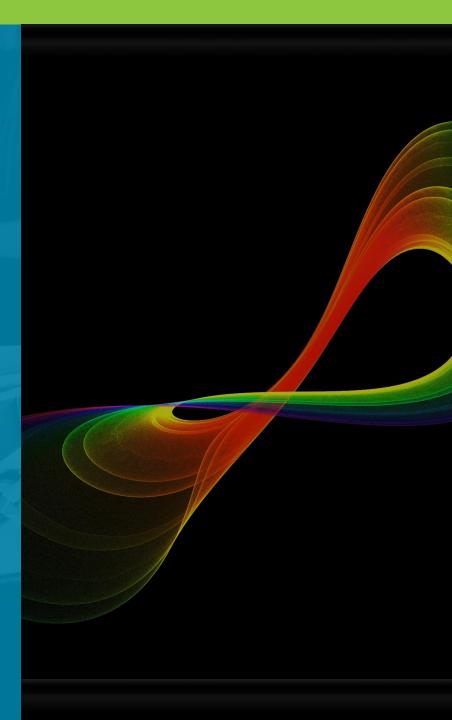
Students must memorize the first 6 perfect cubes (1, 8, 27,..., 216).





Undefined Value Over the Set of Real Numbers





Undefined Value Over the Set of Real Numbers

There are two types of expressions that are undefined over the set of real numbers:

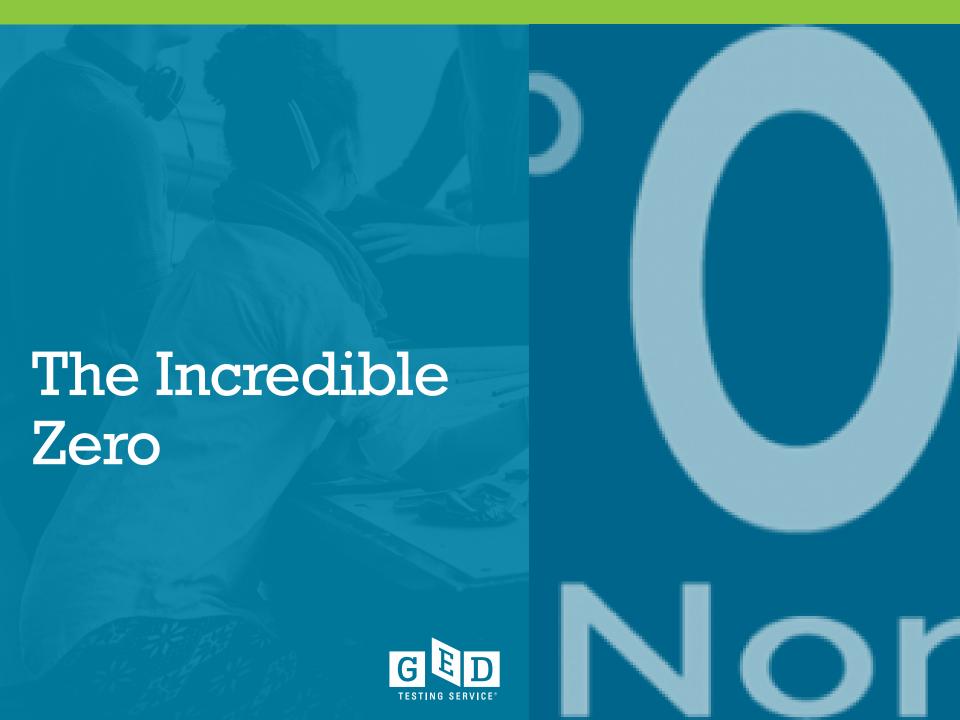
 Fractions with zero in the denominator (or an expression equivalent to zero)

Examples:
$$\frac{-3}{0}$$
; $\frac{0}{0}$; $\frac{x-3}{x+3}$, where $x = -3$

 Square roots of negative numbers (or expressions which, when simplified, result in negative numbers).

Examples:
$$\sqrt{-1}$$
; $x^2 + 1 = 0$; $\sqrt{-3x^2}$; $\sqrt{x^3 - 2}$, where $x = -1$





The Incredible Zero

- It is unique in representing nothingness.
- As a placeholder it gives our number system its power.
- It acquires different meaning based on its location. Think 30 versus 3,000.



The Origin of the Number Zero

http://www.smithsonianmag.com/history/origin-number-zero-180953392/#qagAYijydW3RXhhk.99





Properties of Zero

Property	Example
a + 0 = a	4 + 0 = 4
a - 0 = a	4 - 0 = 4
$a \times 0 = 0$	$6 \times 0 = 0$
0 / a = 0	0/3 = 0
a / 0 = undefined (<u>dividing by zero is undefined</u>)	7/0 = undefined
0 ^a = 0 (a is positive)	$0^4 = 0$
$a^0 = 1$	$7^0 = 1$

http://www.mathsisfun.com/numbers/zero.html





The Problem with Zero





You can express a fraction with 0 in the denominator, but it has no meaning.

Division by zero is undefined. Mathematicians have never defined the meaning because there is no good definition. How many times can you throw nothing into no baskets?

As many times as you want. It's just not a real number.



To learn more:

https://www.youtube.com/watch?v=N KmGVE85GUU



Imaginary Numbers?

Try squaring numbers to see if we can get a negative result.

$$1^2 = 1$$
 $0^2 = 0$ $(-2)^2 = 4$ $(0.2)^2$

"Imagine" there is such a number. Let's call this number *i* for imaginary. Then we can do this...

$$i^2 = i \cdot i = -1$$

"Imagine" there is such a number, called *i* for imaginary. Then we can also do this...

$$\sqrt{i^2} = \sqrt{-1}$$

$$i = \sqrt{-1}$$

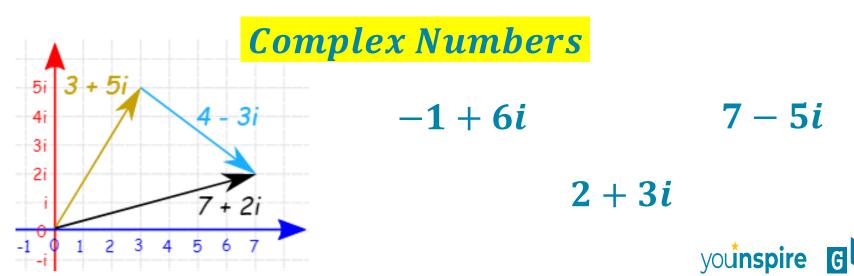




Are imaginary numbers truly imaginary?

There was a point in time when they were thought to be impossible. But then people researched them more and discovered they were actually useful and important because they filled a gap in mathematics (but the word "imaginary" stuck).

The true power of imaginary number comes when combined with real numbers. This gave birth to a whole new mathematics...



Do imaginary numbers serve any purpose?



https://youtu.be/tSamA58MhQ8

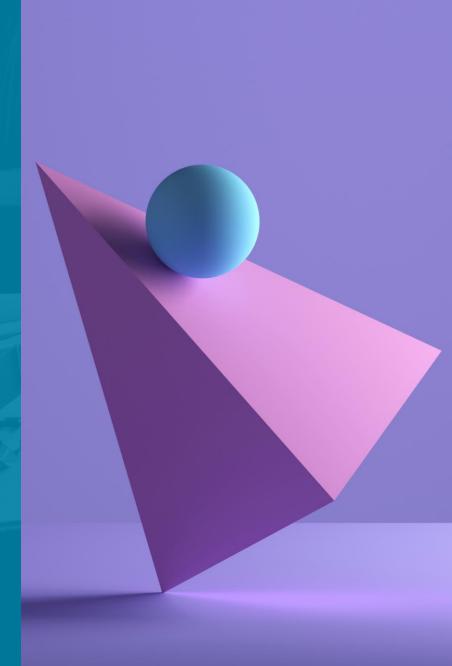


https://youtu.be/fFuVJd36iKg

3-Dimensional Shapes

Performance Gap 3





Volume of a Cylinder

Find the volume of the pizza below.



From the Formula Sheet:

$$V = \pi r^2 h$$

Substitute given information.

$$V = \pi z^2 a$$

Notice anything?

$$V = pi(z \cdot z)a$$

This is why it's called pizza.



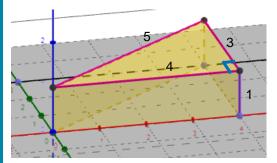
Anchor Chart: Surface Area of Right

Triangular Prisms

$$SA = ph + 2B$$

Workbook P. 7

1 Perimeter of the Base and Height



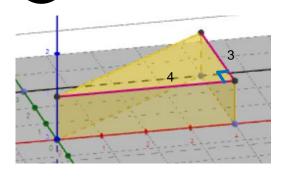
p and h

$$p = (3 + 4 + 5)$$

$$p = 12$$

$$h = 1$$

2 Area of the Base



B

$$B = \frac{1}{2}bh$$

$$B = \frac{1}{2}(3 \cdot 4)$$

$$B = \frac{1}{2}(12) = 6$$

3 Solve

$$SA = ph + 2B$$

$$SA = (12)(1) + 2(6)$$

$$SA = 12 + 12$$

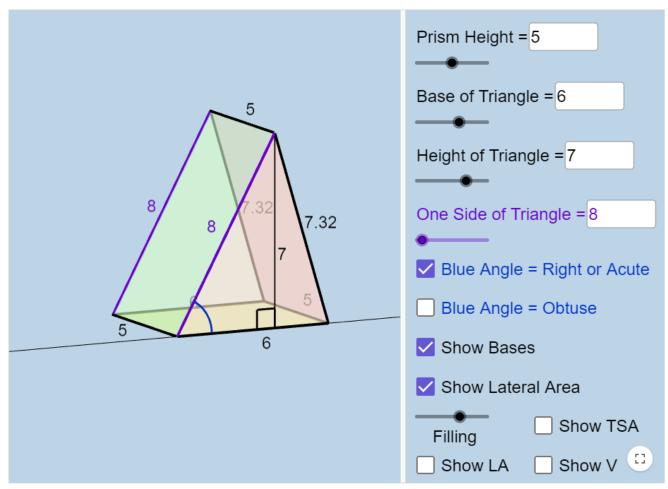
$$SA = 24$$





Examining a Right Triangular Prism







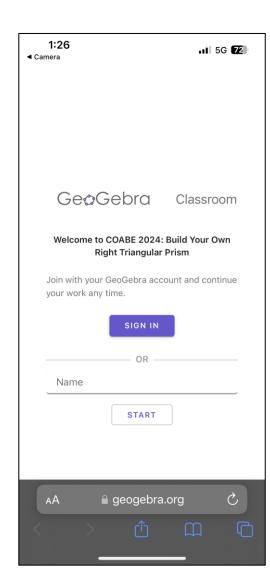


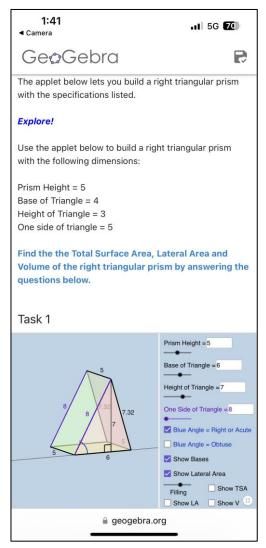
Build Your Own Right Triangular Prism





- 1. Enter your name or a proxy name.
- 2. Tap on "START."
- Follow the instructions to build the right triangular prism.
- 4. Answer the questions that follow.

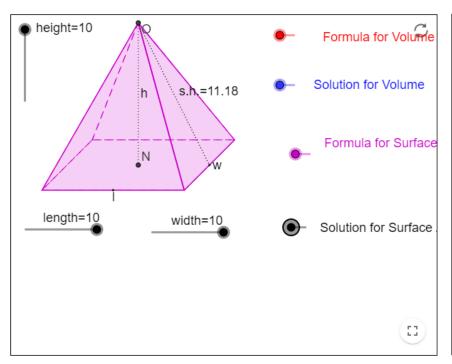


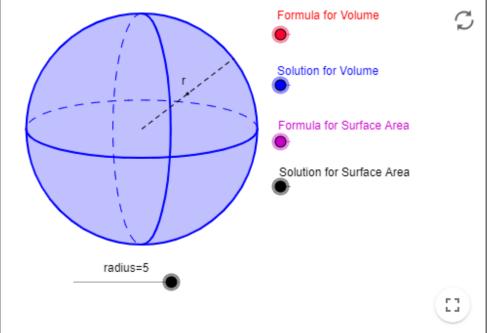


Build and Explore Your Own Solids



There are several other lessons/activities for various solids such as pyramids, cylinders, prisms, cones, and spheres, developed by other teachers and available under Classroom Resources.









GeoGebra's Augmented Reality Feature



















Solving Algebraic Inequalities

Performance Gap 5





Focusing on High Impact Indicators - Inequalities

A.3 Write, manipulate, solve, and graph linear inequalities

A.3.a Solve linear inequalities in one variable with rational number coefficients.

A.3.b Identify or graph the solution to a one variable linear inequality on a number line.

A.3.c Solve real-world problems involving inequalities.

A.3.d Write linear inequalities in one variable to represent context.





Solve Real World Problems with Inequalities

Annie is planning a business meeting for her company. She has a budget of \$1,325 for renting a meeting room at a local hotel and providing lunch. She expects 26 people to attend the meeting. The cost of renting the meeting room is \$270. Write an inequality to show how to find the amount, x, Annie can spend on lunch for each person?



Establish the relationship:

Cost ≤ Budget or Budget ≥ Cost

Budget = \$1,325

Cost = 26x + \$270

Set-up the inequality.

Cost ≤ Budget

 $$26x + $270 \le $1,325$





Solving Equations vs. Inequalities

$$3x + 15 = 24$$

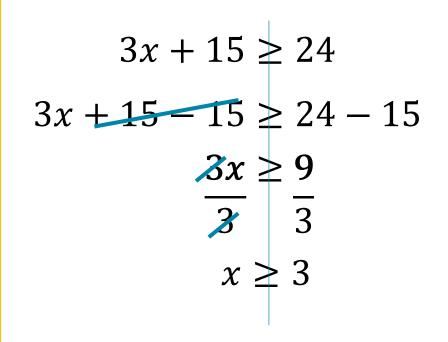
$$3x + 15 - 15 = 24 - 15$$

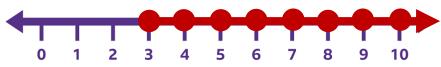
$$3x = 9$$

$$3 = 3$$

$$x = 3$$











With Only One Exception...

When multiplying or dividing both sides of the inequality with a negative number, the inequality sign must be reversed for the solution to remain true.

For example:

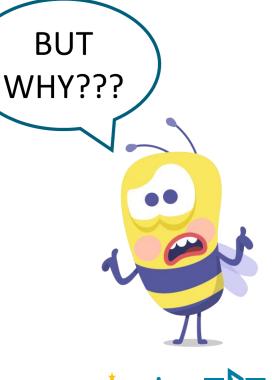
$$-3x - 15 \ge 24$$

$$-3x - 15 + 15 \ge 24 + 15$$

$$-3x \ge 39$$

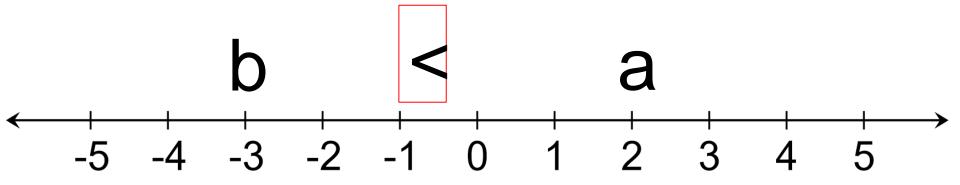
$$\frac{-3x}{2} \le \frac{39}{2}$$

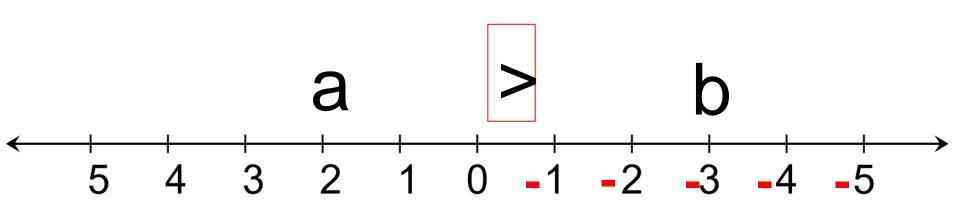






The Million Dollar Question



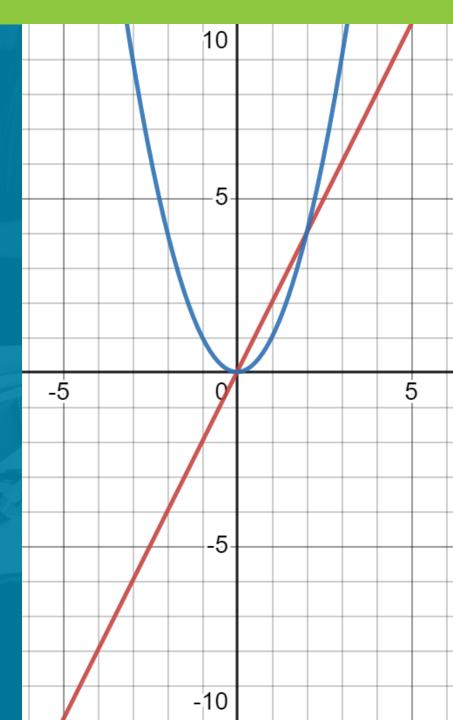






Slopes and Graphing Linear Equations

Performance Gap 6



Focusing on Slopes and Graphing Linear Equations

Slopes and Graphing Linear Equations

Determine the slope of a line from a graph, equation, or table

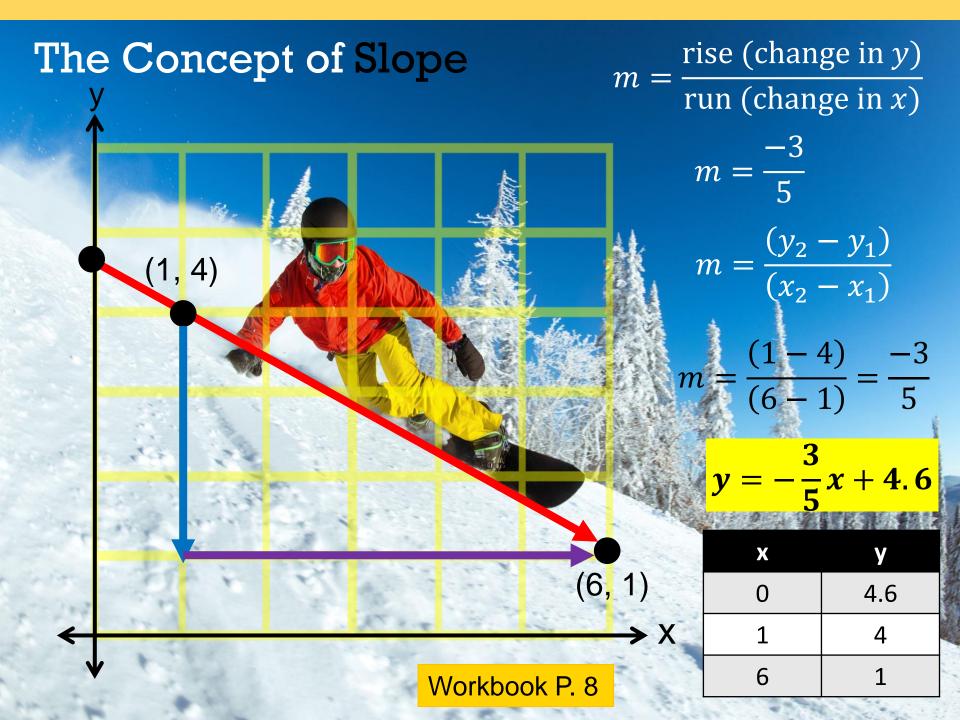
Graph twovariable linear equations. Write the equation of a line with a given slope through a given point.

Use slope to identify parallel and perpendicular lines and to solve geometric problems.

Write the equation of a line passing through two given distinct points.









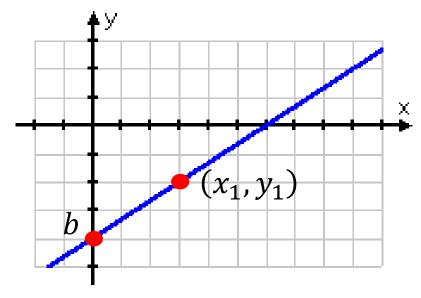




Forms of Linear Equations

Forms of Linear Equations	Equations
Slope-Intercept Form	y = mx + b
Point-Slope Form	$y - y_1 = m(x - x_1)$
Standard Form	cx + dy = e

m = slope b = y - intercept $(x_1, y_1) = \text{a point on the line}$ c, d and e are constants



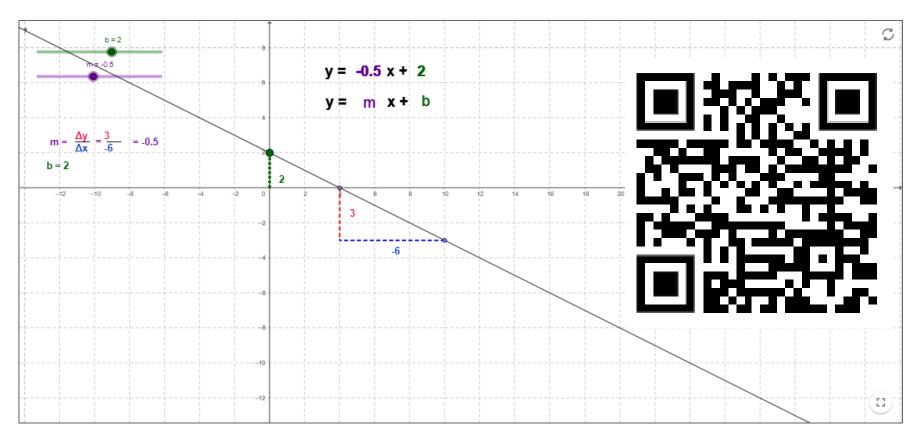
Workbook P. 8





Graphing Linear Equations





https://www.geogebra.org/m/n5gskda8





Anchor Chart for Finding Slope

T-Chart	Slope-Intercept	Standard	Graph
Use the slope formula. $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$	Locate m in the equation. $y = mx + b$	cx + dy = e Transform equation to slope-intercept form and locate m in the equation.	3 2 1 1 2 3
x y 1 -9 3 -6 5 -3	Example: $y = 3x - 4$ $y = mx + b$	Example: 3x + 9y = 4 $-3x - 3x$ $9y = -3x + 4$ $9y -3x + 4$	Locate two points on the graph, then use the slope formula. Example: (0.2) and(2,3)
$m = \frac{-3 - (-9)}{5 - 1}$ $m = \frac{6}{4} = \frac{3}{2}$	m = 3	$\frac{9y}{9} = \frac{-3x}{9} + \frac{4}{9}$ $y = \frac{-3}{9}x + \frac{4}{9}$ $m = \frac{-3}{9}$	$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ $m = \frac{3 - 2}{2 - 0}$ $m = \frac{1}{2}$

Graphing Exercise



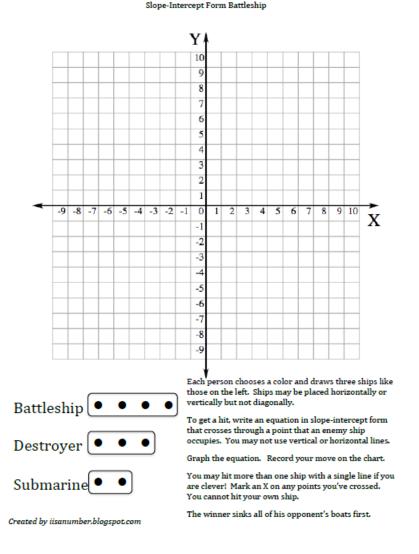
1 ∅ 22% T-Mobile 🛜 4:11 PM nearpod **Students** Nearpod App Enter lesson CODE **Teachers** New to Nearpod? **Get Started Now** Already have an account? Sign In

Join at join.nearpod.com or in the app

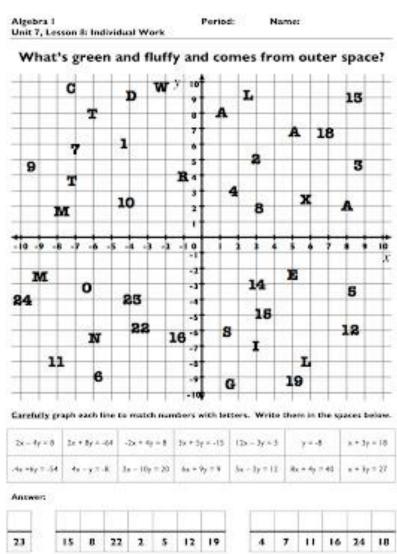




Supplemental Graphing Activities



Kathryn (2013). Slope-Intercept Form Battleship. i is a number. http://iisanumber.blogspot.com/2013/02/slope-intercept-form-battleship.html



D. Wekselgreene (2010). Some fun(ish) worksheets. http://exponentialcurve.blogspot.com/2010/04/some-funish-worksheets.html

Anchor Chart for Using Slope to Interpret Distance vs. Time Graphs

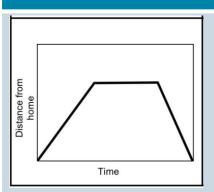
Scenario 1

Distance from home

Time

Scenario 2

Scenario 3 Scenario 4



Example:

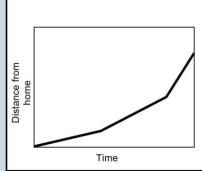
Maria walked to the store at the end of her street, bought a gallon on milk and then ran all the way back.

 Γime	Distance	
0	0	
1	20	
2	40	
3	40	
4	40	
5	0	



Lucy walked slowly along the road, stopped to look at her cell phone, realized that she was late, and then started running.

Time	Distance
0	0
1	20
2	40
3	40
4	80
5	120



Example:

Opposite Tom's home is a hill. He climbed slowly up the hill, walked across the top, and then ran quickly down the other side.

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

Distance from home

Time

Example:

Mario went out to walk with some friends. Upon realizing he left his wallet, ran back home to get it. He then had to run to catch up with the others.

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

Linear Modeling Word Problem

Mathematical Reasoning

Question 6 of 10

✓ Answer Explanation

☐ Calculator

A Flag for Review

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.

Tree 1

1166 1		
Year	Trunk Diameter (inches)	
1	18.6	
3	19.2	
5	19.8	
7	20.4	
9	21.0	
11	21.6	
13	22.2	

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(19.2 - 18.6)}{(3 - 1)}$$

$$m = \frac{(0.6)}{(2)}$$

$$m = 0.3$$

$$y = mx + b$$

$$18.6 = 0.3(1) + b$$

$$18.6 - 0.3 = b$$

This is the final year in which she will collect data. When her data collection is complete, she will predict future red maple tree growth.

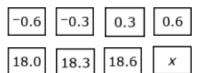
■ Formula Sheet

☐\ Calculator Reference

The scientist creates an equation that models her data for each tree so that she can predict the diameter in the future. Complete a linear equations that fits the data for tree 1, where x is the year and y is the trunk diameter, in inches.

Click on the variables and number you want to select and drag them into the boxes.

$$y = mx + b$$
Equation for Tree 1
$$y = y = y + y$$



Linear Modeling Word Problem

Mathematical Reasoning

Question 6 of 10

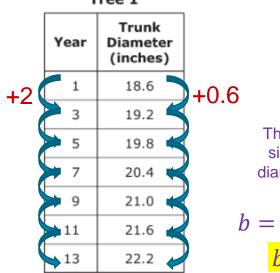
✓ Answer Explanation

☐ Calculator

A Flag for Review

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.

Tree 1



$$m = \frac{0.6}{2}$$

$$m = 0.3$$

The y-intercept is simply the trunk diameter at year 0.

$$b = 18.6 - 0.3$$

$$b = 18.3$$

This is the final year in which she will collect data. When her data collection is complete, she will predict future red maple tree growth.

■ Formula Sheet

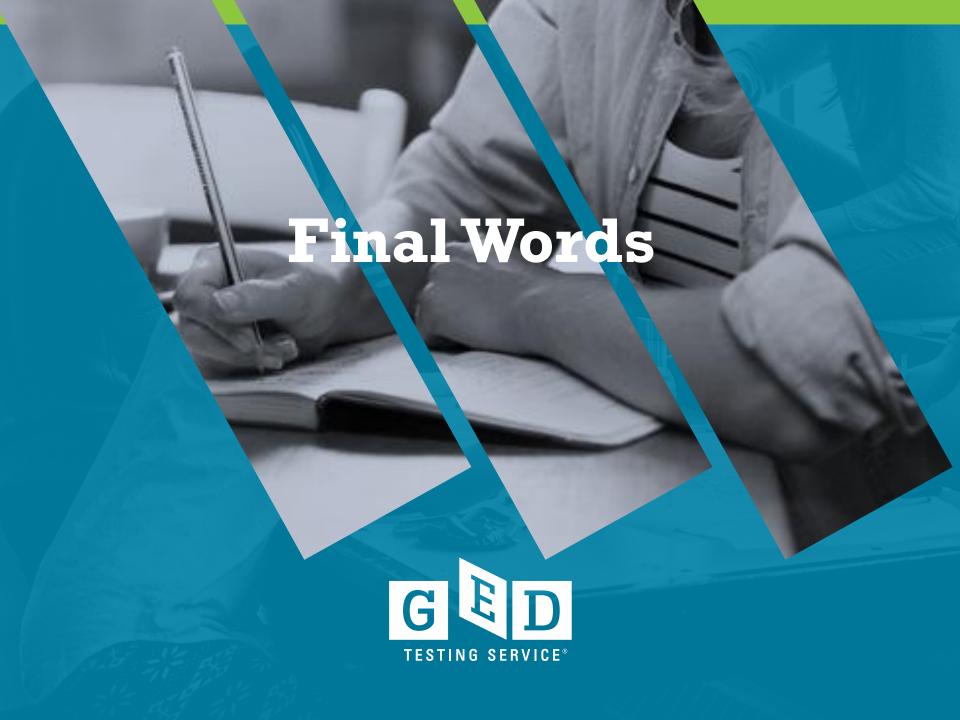
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The scientist creates an equation that models her data for each tree so that she can predict the diameter in the future. Complete a linear equations that fits the data for tree 1, where x is the year and y is the trunk diameter, in inches.

Click on the variables and number you want to select and drag them into the boxes.

$$y = mx + b$$

0.3 0.6





66

Some people want it to happen, some wish it would happen, others make it happen.



Michael Jordan





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