



Developing GED® Test Geometric Reasoning Skills

COABE 2018

Session Objectives

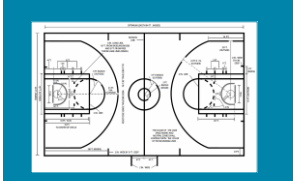
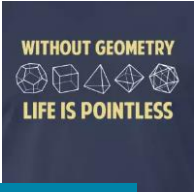


- Explore strategies and activities to improve students’ geometric reasoning skills
- Practice learning activities
- Explore resources

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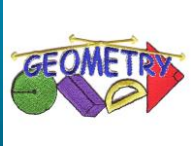
Geometry – It’s All Around Us!



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Building Geometric Reasoning Skills



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Assessment Targets—Q.4/Q.5

- **Q.4** Calculate dimensions, perimeter, circumference, and area of two-dimensional figures
- **Q.5** Calculate dimensions, surface area, and volume of three-dimensional figures

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What to look for in student work (Q.4/5)

The student can

- identify the dimensions of a geometric figure from a diagram, then substitute the values for those dimensions into the appropriate formula for geometric measurement, then calculate the resulting numerical expression.
- calculate the perimeter of polygons.
- identify the shapes that comprise a composite figure.

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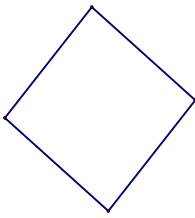
Van Hiele Theory

- Level 1: Visualization
- Level 2: Analyze
- Level 3: Informal Deduction
- Level 4: Formal Deduction
- Level 5: Rigor

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Visualization



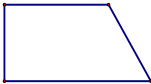
- Recognize and name shapes by appearance
- Do not recognize properties or if they do, do not use them for sorting or recognition
- May not recognize shape in different orientation (e.g., shape at right not recognized as square)

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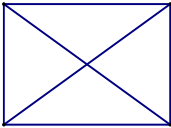
Analysis



- Can identify some properties of shapes
- Use appropriate vocabulary
- Cannot explain relationship between shape and properties (e.g., why is second shape not a rectangle?)

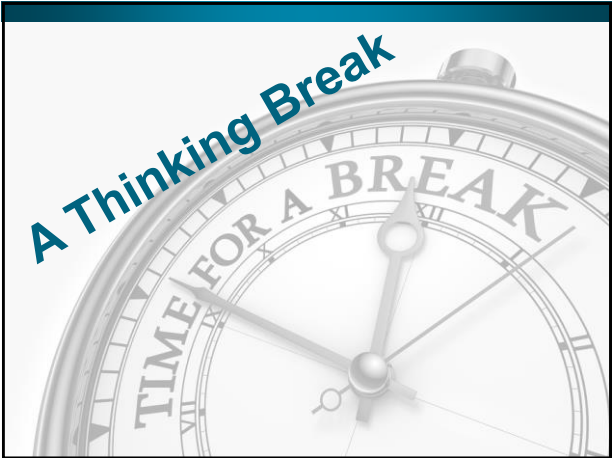
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Analysis – Implications for Instruction



- Work with manipulatives
- Define properties, make measurements, and look for patterns
- Explore what happens if a measurement or property is changed
- Discuss what defines a shape
- Use activities that emphasize classes of shapes and their properties
- Classify shapes based on lists of properties

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Triangle

Congruent

Quadrilateral

Polynomial

Rotation

Right Angle

Reflection

Parallel

Use the Language
of Geometry!
It's Geometry Match!

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Example of a Vocabulary Strategy

Definition (in own words)

Facts/Characteristics

WORD or SYMBOL

Examples

Non-Examples

Frayer Model – (Barton and Heidema, 2002)

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Example of a Vocabulary Strategy

Definition

Facts/Characteristics

Equation

Examples

Non-examples

An equation is a mathematical statement that shows that two expressions are equal.

- always has exactly one equal sign
- the left side is equivalent to the right side
- some equations have 0, 1, 2 or more solutions
- some equations contain just numbers
- some equations are algebraic models for relationships and they have corresponding graphical models and numerical models (e.g., tables)

$3x - 2 = 4x + 7$ (linear equation)
 $ab = ba$ (an identity)
 $F = 1.8C + 32$ (a formula)
 $5 + 6 = 11$ (a number statement)
 $P = 2l + 2w$ (a formula)
 $x = 3$ (statement of value)

$2x + 3y$ (expression)
3 (number)
perimeter (word)
 $x < y$ (inequality)
 $= 4.2$ (has no left side)

Frayer Model – (Barton and Heidema, 2002)

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Use Manipulatives

Get Students Thinking!

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Tangrams

- Originated in China
- Its invention is unrecorded in history.
- The word “tangram” may have come from the Tanka people who were traders who played this puzzle and past it on via sailors.
- The word might come from an obscure English word “trangram” which means puzzle or trinket.
- Became popular during the 19th century in Europe and America.



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Tangrams

Tangram puzzles are made of seven shapes:

- 5 triangles
- 1 square
- 1 parallelogram

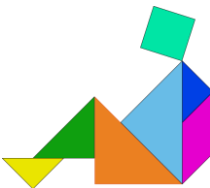


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Tangrams

Teach math in an interesting manner

- Recreate given shapes
- Create new shapes
- Integrate mathematical skills using manipulatives (e.g., What fraction of the original shape are each of the 7 pieces?)

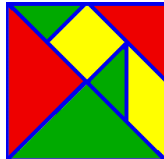


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Tangrams

Reinforce geometric principles and ratios

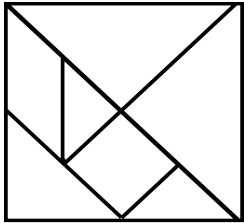
- Given a specified side length, find the area of each of the smaller shapes making up the tangram.
- Determine different ways all 7 shapes be used to make a square.
- Given a specified side length, determine individual side lengths (and/or perimeters) of each of the 7 pieces.
- Find the smallest perimeter using all 7 shapes. The maximum perimeter.



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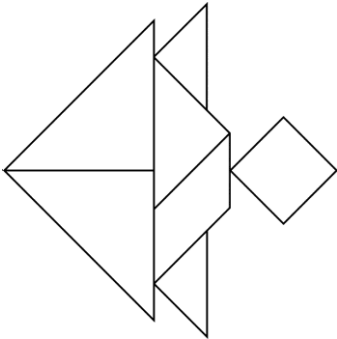
Tangrams

Can you make a square using all of your tangram pieces?



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Create the Shape – An Angel Fish



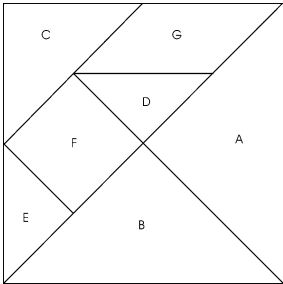
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Tangrams

Sample problem

- If the entire tangram is worth \$120, how much is each piece worth?

What skills will a student use with this problem?



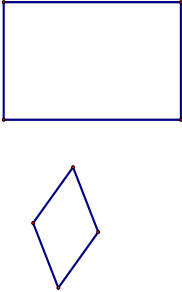
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Informal to Formal Deduction

The Third and Fourth Steps in Geometric Reasoning

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Informal Deduction

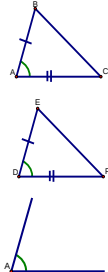


- Can see relationships of properties within shapes
- Recognize interrelationships among shapes or classes of shapes (e.g., where does a rhombus fit among all quadrilaterals?)
- Can follow informal proofs (e.g., every square is a rhombus because all sides are congruent)

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Deduction



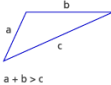
- Usually not reached before high school; maybe not until college
- Can construct proofs
- Understand the importance of deduction
- Understand how postulates, axioms, and definitions are used in proofs

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It's All About Geometric Reasoning

Theorem: The measure of any side of a triangle must be LESS THAN the sum of the measures of the other two sides. (This same concept forms the basis for other questions in the domain of Geometry.)



The Triangle Inequality Theorem is the mathematical statement of the old adage, "The shortest distance between two points is a straight line." If you don't travel along the straight line, you travel two sides of a triangle, and that trip takes longer.

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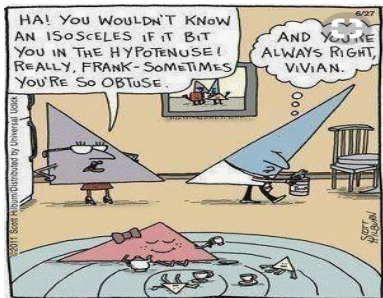


Some say it's all about TRIANGLES!

Theorem: The measure of any side of a triangle must be LESS THAN the sum of the measures of the other two sides. (This same concept forms the basis for other questions in the domain of Geometry.)

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Your Task!

*How many different triangles can you make with 11 toothpicks?
(using a whole number of toothpicks per side and all 11 sticks for each triangle)*

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It's All About Triangles

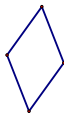
How many different triangles can you make with 12 toothpicks?

- Surprisingly the count goes down if you add another matchstick to the mix: With 12 matchsticks one can only make **three** different triangles: 5-5-2; 5-4-3; 4-4-4.
- What's going on?

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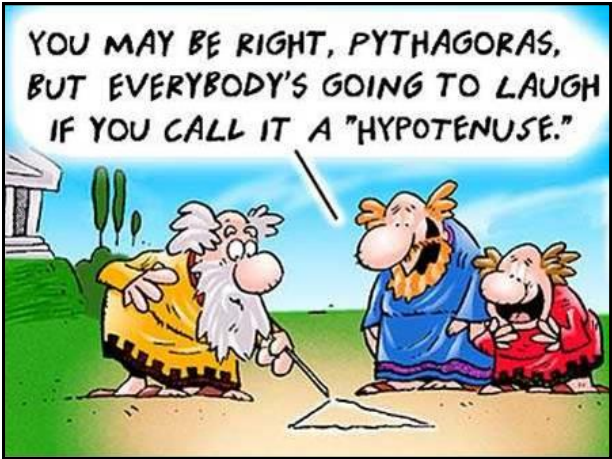
Informal Deduction



- Can see relationships of properties within shapes
- Can recognize interrelationships among shapes or classes of shapes (e.g., where does a rhombus fit among all quadrilaterals?)
- Can follow informal proofs (e.g., every square is a rhombus because all sides are congruent)

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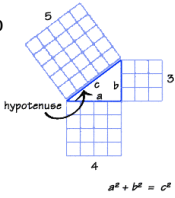




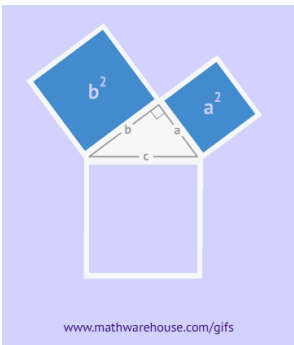
What Does This Have to Do with Anything?

The *Pythagorean Theorem* states
"The sum of the squares of the lengths of the legs of a right triangle ('a' and 'b' in the triangle shown below) is equal to the square of the length of the hypotenuse ('c')."

$$a^2 + b^2 = c^2$$



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www.mathwarehouse.com/gifs

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Have You Ever Used the
Pythagorean Theorem?

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Pythagorean in Real-Life

A customer would like a bonus room to be added to an existing home. The new room is to be 26' x 24' with an 8' ceiling and a 2' roof overhang. The ridge of the roof is to be centered over the 24 foot wall and 5 feet above the top of the wall of the bonus room. Assuming the builder uses standard 4' x 8' plywood sheets, determine the following:

How many plywood sheets will be needed to cover the walls of the bonus room (not accounting for doors or windows)?

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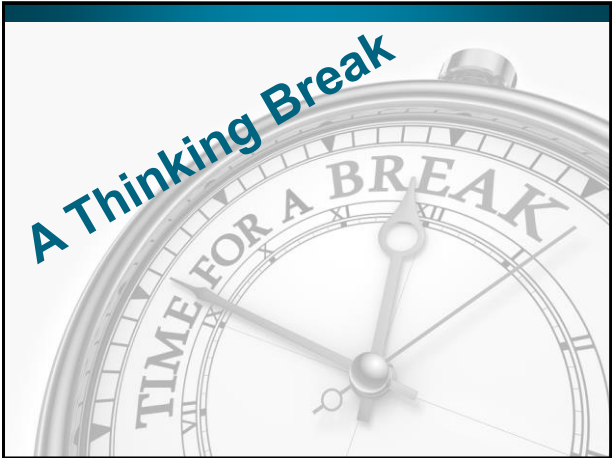
Pythagorean in Real-Life

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Divide the area of the walls by the area of plywood sheet
26' x 8' (2 walls) + 24' x 8' (2 walls)
4' x 8' plywood sheet

$(2 \times 208 \text{ sq. ft.}) + (2 \times 192 \text{ sq. ft.}) / 32 \text{ sq. ft.} =$
 $416 \text{ sq. ft.} + 384 \text{ sq. ft.} / 32 \text{ sq. ft.} =$
 $800 \text{ sq. ft.} / 32 \text{ sq. ft.} =$
25 sheets of plywood for the walls

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GETTING A GRIP ON SURFACE AREA AND VOLUME

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Formulas

Figure	SA Formula	V Formula
Rectangular/right prism	$SA = ph + 2B$	$V = Bh$
Cylinder	$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
Pyramid	$SA = \frac{1}{2}ps + B$	$V = \frac{1}{3}Bh$
Cone	$SA = \pi rs + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$
Sphere	$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

p = perimeter of base with area B; $\pi = 3.14$

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Two Ways to Do It

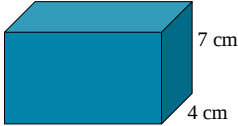
- Surface area can be computer by
 - using the formula or
 - finding the area of each surface and adding them up!

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Example:



Surface Area

Top/bottom $2(8)(4) = 64$

Left/right $2(4)(7) = 56$

Front/back $2(8)(7) = 112$

Add them up!

$SA = 232 \text{ cm}^2$

$V = lwh$

$V = 8(4)(7)$

$V = 224 \text{ cm}^3$

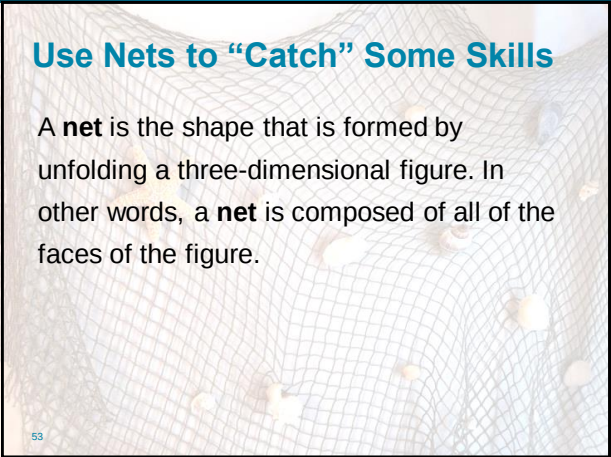
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Use Nets to “Catch” Some Skills

A **net** is the shape that is formed by unfolding a three-dimensional figure. In other words, a **net** is composed of all of the faces of the figure.

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Exploring Surface Area, Volume and Nets

Math Interactives

Use It 1: Nets

Use It 2: Views

Find the nets for all 6 objects.

The slider provides an animation between the rectangular prism and its net. After watching the animations several times, select the appropriate net below:

http://www.learnalberta.ca/content/mjg/hm/index.htm?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SURF&less on=html/object_interactives/surfaceAreaUse_it.html

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Using Nets to find Surface Area

Find the surface area of the rectangular prism by using a net.

Graph paper is included in your workbook.

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Geometry Challenges

Mathematical Reasoning - Candidate Name

Answer Explanation

Calculator

Question 10 of 10

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.

Tree 1		Tree 2	
Year	Trunk Diameter (inches)	Year	Trunk Diameter (inches)
2002	12	2002	10
2004	14	2004	12
2006	16	2006	14
2008	18	2008	16
2010	20	2010	18
2012	22	2012	20

Tree Wrap

The wrap is priced by the square foot. To the nearest square foot, how many square feet of wrap does she need?

☐ A. 22

☐ B. 44

☐ C. 121

☐ D. 261

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Interactives Geometry 3D Shapes

Annenberg Learner

- <http://www.learner.org/interactives/geometry/index.html>

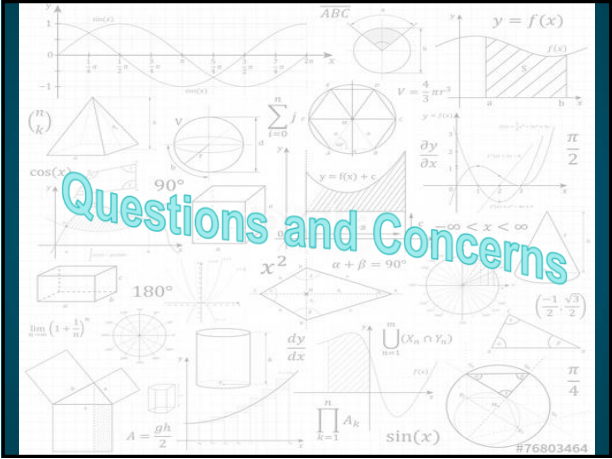
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CHALLENGES ARE GIFTS THAT FORCE US TO SEARCH FOR A NEW CENTER OF GRAVITY. DON'T FIGHT THEM. JUST FIND A NEW WAY TO STAND

- OPRAH WINFREY





Thank you!

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